Journal of Artificial Intelligence and Data Mining (JAIDM), Vol. 13, No. 1, 2025, 75-84.

Shahrood University of Technology

Journal of Artificial Intelligence and Data Mining (JAIDM)
Journal homepage: http://jad.shahroodut.ac.ir

Original/Review Paper

Hybrid Image Inpainting: Combining Low-rank Minimization and Spline-based Approach

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Article Info

Article History:
Received 07 July 2024
Revised 17 November 2024
Accepted 15 January 2025

DOI:10.22044/jadm.2025.14744.2576

Keywords:
Target Region, Spline, Low-rank Minimization, Tensor.

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Abstract

Image inpainting is one of the important topics in the field of image processing, and various methods have been proposed in this area. However, this problem still faces multiple challenges, as an inpainting algorithm may perform well for a specific class of images but may have poor performance for other images. In this paper, we attempt to decompose the image into a low-rank component and a sparse component using (Principal Component Analysis) PCA, and then independently restore each component. For inpainting the low-rank component, we use an algorithm based on low-rank minimization, and for restoring the sparse component, we use the concept of splines. Using splines, we can effectively restore edges and lines, whereas the restoration of these regions is challenging in most algorithms. Also, in restoring the low-rank component, we construct a tensor at each step and approximate the missing pixels in the tensor, thereby significantly improving the efficiency of the low-rank minimization idea in image inpainting. Finally, we have applied our proposed method to restore various types of images, which demonstrates the effectiveness of our proposed method compared to other inpainting methods based on PSNR and SSIM.

1. Introduction

Image inpainting is a problem in the field of image processing, aimed at completing the missing pixel, corrupted, or parts of an image that are intended to be removed, such as removing a specific object from the image. These regions can be logos, subtitles, text, or any other objects in the image that are considered undesirable by the user, and they are referred to as the "target region" or generally as "holes" in the inpainting problem. It is noticeable that, they may also be called the "removed region" or "occluded object". The inputs to the inpainting problem include the original image and a mask image, where the user specifies the regions that need to be inpainted. The estimation of the missing pixels is done using the source regions of the image, known as the "source region", and this process is called image inpainting or image restoration. This term is derived from the process that restoration artists perform to hide the damages and injuries that have occurred in their paintings.

The first research in the field of image inpainting was presented in 2000 by Bertalmio et al. [1], where they used partial differential equations (PDE) for inpainting. The main idea of this method was to propagate the boundary pixels of the corrupted region to restore the image. After that, numerous algorithms have been proposed to improve image inpainting, each with its own strengths and weaknesses. Generally, the existing methods in image inpainting either work pixel-by-pixel or in a patch-by-patch manner. Additionally, in another classification, image inpainting algorithms can be divided into patch-based methods, diffusion-based methods, and matrix/tensor low-rank minimization algorithms, which will be further explained. Formally, in an inpainting problem, a region Ω of the given image has unknown pixel values, and the goal is to find appropriate values for this region such that the inpainted image appears natural. It is important to note that the region Ω is provided as an input to the inpainting problem. More precisely, a binary mask with the same dimensions as the original image is available, where the pixels belonging to the target region Ω are marked with a different color from the source region Φ, for example, the target region pixels are shown in white and the source region in black.

2. Related work

Image inpainting methods are categorized into three classes as the patch-based method, diffusion-based method, rank minimization algorithm. In recent years, patch-based algorithms and rank minimization have received more attention, and diffusion-based methods are less frequently mentioned [25]. It should be mentioned that it is a database related to Berkeley University [2], which some articles on image restoration have used to evaluate their method. However, some articles have used arbitrary images for testing the performance of their methods.

2.1. Patch-based Methods

The patch-based algorithms are considered the most widely used image inpainting methods. This category was first introduced by Criminisi et al. [3], and subsequent works have been done in this area [4, 5, 6, 7]. The core idea of these methods is texture synthesis [8], where similar patches can be used to fill the unknown region. In this category, patches are considered at the boundary between the target and known regions, where some pixels are known, and some are unknown. These patches are centered at a point P and denoted as φ(P). Then, a search is performed across the entire image to find the most similar patch to φ(P), denoted as φ(Q), based on a similarity metric provided by the algorithm. After finding the most similar patch, the unknown pixels in φ(P) are filled with the corresponding pixels from φ(Q), and this process continues until all the pixels in the target region are filled.

Another patch-based method proposed by Xu et al. [5] introduces a new priority for filling the pixels. According to this method, the more unique a patch is in the boundary region, the higher priority it has for filling. The uniqueness of a patch is determined by the number of similar patches in its neighborhood, which is referred to as the "sparsity" criterion. This method also suggests selecting multiple candidate patches for each target patch, instead of just one. In another patch-based approach, in addition to the challenge of determining the filling priority, the similarity metric between two patches is also addressed [9], so that it can be said that this method is based on the patch with the approach of preserving the structure of the image and texture. As a measure of similarity, in addition to the color features between two patches, the spatial distance between two patches is also considered [9]. In this method, a new criterion for the priority of patches has been proposed, in which patches with more structural complexity have a higher priority.

2.2. Diffusion-based Inpainting Methods

Diffusion-based methods were first introduced by Bertalmio [1]. For improving diffusion-based algorithm, the gradient space for information diffusion was used [10] and minimization based on the Euler's scheme for repairing curve structures [11] can be mentioned. One weakness of this class of methods is that the restored edges tend to have a somewhat blurred appearance. Another point to note is that diffusion-based methods are not well-suited for efficiently propagating textures, and they work better for small and narrow target regions, while they can create blurriness for large target regions. Although, a deblurring method can be used to obtain a higher quality from a degraded image. A deblurring algorithm was proposed by Sahragard et al. [31] based on the total variation regularizer. The variable of optimization problem is split, and the new optimization problem is solved by using Lagrangian augmented method.

Another method, a novel approach by Ballester et al. [12] involved introducing a new partial differential equation (PDE) formula for filling the target region, with the aim of enhancing the previous method. Later, another method was proposed that performed inpainting based on diffusion in the gradient space, which led to a significant improvement in the image quality after inpainting [10]. Another algorithm in this domain was presented by Chan and Shen, which was based on the improved Curve Completion Diffusion (CCD) [16]. Additionally, two more algorithms were proposed by the same authors, one based on TV minimization [11] and the other using the Euler's scheme for propagating curve structures [13].

2.3. Matrix/Tensor Low-Rank Minimization Inpainting Methods

To mathematically formulate the inpainting problem, the given image is considered as a matrix D, where some of the data in this matrix are missing. The goal is to estimate the unknown data based on the available data, and the resulting matrix X is the inpainted image. The available data in the matrix can be considered as the source region in the image. Unlike patch-based and diffusion-based algorithms, in this category of methods, the symbol Ω is used to refer to the source region (or the known pixels). Based on the given explanation, an image inpainting problem can be modeled as the relationship in Equation (1) [14]:

$$\min(rank(X)) \tag{1}$$

So that for ∀(i,j)∈Ω the relationship X(i,j)=D(i,j) is established or the difference between these two parameters Ω is less than a threshold limit. Also, rank(X) means the order of X matrix. That is, the unknown values of matrix D are approximated and the result is called matrix X. This approximation should be done in such a way that the order of matrix X is minimized, as well as the pixels of matrix X that are in the position corresponding to the pixels of the source area, to be as similar as possible to the pixels of the matrix D from the source area. The problem of minimizing the matrix rank is known as an NP-problem [14]. So, it needs to be simplified to be solvable. It is said that using the nuclear norm NN[1] is the best substitute for minimizing the matrix rank [14]. Therefore, instead of Equation (1), Equation (2), which is known as the Nuclear Norm Minimization NNM[2] problem, should be minimized:

$$\min(\|X\|_*) \tag{2}$$

where ‖X‖* denotes the nuclear norm of matrix X, which is defined in Equation (3). Both in Equation (1) and Equation (2), the expectation is that for all $\forall(i,j) \in \Omega$, the relationship $X(i,j) = D(i,j)$ holds.

$$\|X\|_* = \sum_{i=1}^{r} \sigma_i \tag{3}$$

where σi represents the i-th singular value (or singular point), in other words, the nuclear norm of a matrix refers to the sum of its singular values. Several algorithms have been proposed to solve the optimization problem in Equation (2). Based on the discussed information, the goal is to minimize the rank of matrix X, such that the pixel values in the known region are as similar as possible to the pixel values after the matrix rank minimization. The complete formula is given in Equation (4):

$$\hat{X} = \arg\min_{X \in R^{m\times n}} \|X\|_* \quad s.t. \rho_\Omega(X) = \rho_\Omega(D) \tag{4}$$

where D is the observed (or corrupted) matrix that is incomplete and needs to be repaired, and ρ_Ω is a simple operator in the region Ω, whose operation is described in Equation (5).

$$[\rho_\Omega(X)]_{ij} = \begin{cases} X_{ij} & if\,(i,j) \in \Omega \\ 0 & otherwise \end{cases} \tag{5}$$

This equation can be solved using various optimization algorithms such as SVT[3] [15].

It is worth noting that in another classification, there are image inpainting algorithms based on neural networks, which have been widely studied in recent years [29]. For example, a two-stage method has been proposed that uses feature fusion for inpainting. In this approach, a dynamic memory networks (DMN) is used to combine internal and external features from the incomplete image, resulting in an optimized map that needs to be completed. Then, a generative network with gradient penalty is used to fill the missing regions [21]. Another deep learning-based method uses a multi-scale approach to reduce the missing data during convolution, paying attention to both texture and semantic features of the image [22]. In this method, the texture features of the image as well as the semantic features of the image have been paid attention to separately in the restoration process. In another method based on deep learning, an iterative method with feedback is used for image restoration [23]. For this purpose, CAM[4] generalization is used to create high-resolution patches. Corneanu et al. presented another algorithm, that does not require expensive training phase [30]. In this method, the forward-backward fusion step is performed on a latent space rather than the image space. It should be noted that an unconditional diffusion model is applied for generalization of any mask shape for inpainting [30]. Generally, neural network-based methods require training and testing, and therefore are not comparable in terms of execution time and training

[1] Nuclear Norm
[2] Nuclear Norm Minimization
[3] Singular Value Thresholding
[4] Contextual Attention Module

data requirements to other methods that can perform reconstruction solely based on the information in a single image. In our proposed approach, we can perform the reconstruction using only a single image, without the need for multiple images from the same domain and extensive training and testing process.

Overall, the use of low-rank minimization methods may not produce satisfactory results in many images [17]. It is even possible that the initial image does not have the condition of low rank. In our proposed method, we first extract the low-rank component of the image, and then apply the low-rank minimization algorithm to this component. Additionally, one of the important challenges in the field of image inpainting is the repair of curves, edges, and similar regions, which many algorithms struggle with. However, in our method, by extracting the sparse component and using the spline idea, we can effectively repair these target regions. In general, the idea of using two-stage methods has received a lot of attention in recent years [24], and it seems that instead of applying the inpainting algorithm in a single stage, which can produce unsatisfactory results for some images, we can leverage this two-stage approach.

3. Proposed Method

As mentioned, various methods have been proposed for image inpainting, but these methods cannot perform well for all images. One class of image inpainting algorithms is based on minimizing the matrix or tensor rank. Although this class presents a powerful approach for image inpainting, employing them comes with challenges. One of these challenges is the assumption that an image has a low rank.In matrix and tensor rank minimization-based inpainting methods, the image is considered as a matrix or tensor. In the simplest case, a grayscale image is given, where the pixel values correspond to a matrix, and some of the matrix values are unknown. The goal is to estimate the unknown matrix values based on the known values, such that the resulting matrix has a low rank. The concept of low-rank matrices has received significant attention in recent years and has also been applied in the field of image processing. It is argued that natural images may have relatively low-rank structures. Based on this assumption, image inpainting methods based on rank minimization have been proposed.

However, the assumption of low rank may not hold for all images. Therefore, in this paper, we attempt to separate the low-rank component of the image and perform inpainting on this component based on the rank minimization concept. This helps us utilize the power of the rank minimization idea for inpainting all types of images.

To separate the low-rank component of the image, we need an image analysis method, where we have used the concept of PCA[5] to analyze the image and find the structure and texture components. The result of using PCA is two separate images, one of which is low-order and the other sparse, so we consider the low-order image as the texture image and the sparse representation as the structure image. Then each of these components is repaired according to our proposed method and the resulting image is obtained by combining the two repaired images. In Figure 1, the flowchart of our proposed method is attached.

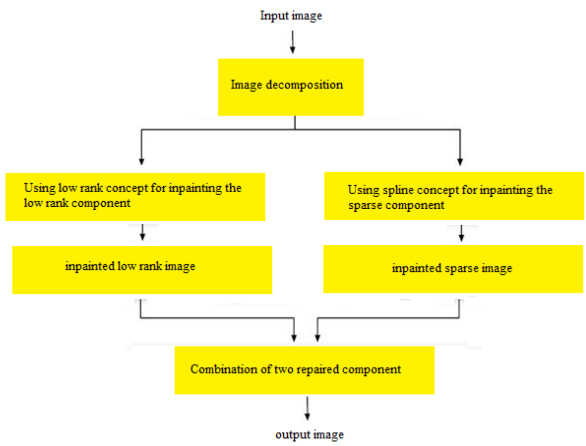

Figure 1. Flowchart of the proposed method.

3.1. Image Decomposition using PCA

PCA can be used for data analysis and pattern recognition, as it maps the data from a higher-dimensional space to a lower-dimensional space. One of the applications of PCA is in the field of image processing, where applying PCA on an image can decompose it into a low-rank component and a sparse component, as shown in Equation (6):

$$I = L + S \tag{6}$$

[5] Principle Component Analysis

where I is the original image, L is the low-rank component, and S is the sparse component. For further explanation, an example of applying PCA and decomposing a color image is shown in Figure 2, and a grayscale image decomposition example is also provided.

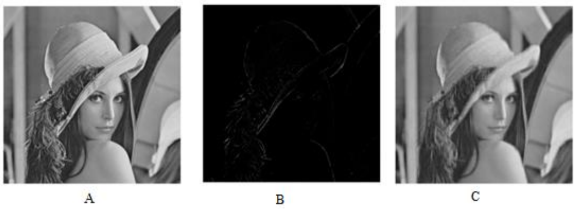

Figure 2. Decomposition of a gray image based on PCA, (A) Original image (B) Image of sparse component (structure), (C) Image of low rank component (texture).

3.2. Sparse Component Inpainting

Generally, one of the weaknesses of various inpainting methods is in repairing curves and objects. When the target region is a part of a curve or an object, using different image inpainting methods cannot recover the overall shape of the curve, and the resulting shape may not be visually pleasing to the human visual system. Therefore, our proposed method is to first perform curve inpainting at the structural image level, and then based on the recovered curve in the target region, perform the final inpainting. Our proposed method is to perform curve inpainting using the concept of splines. It is important to note that in detecting curves using splines, the size of the hole should not be too large. A key point about using splines for inpainting is that splines are applied to the sparse image, not the original image. Given that the sparse image lacks details and texture, using splines can produce very good results. As shown in Figure 3, if we can perform curve inpainting using splines, we can complete the lines and curves present in the image.

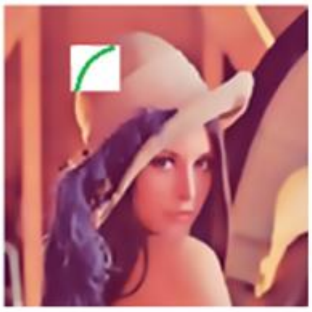

Figure 3. Using spline techniques to repair the curve in the target area

One of the goals we pursue is to first repair objects and curves, which is part of the image inpainting process. This curve inpainting helps preserve the continuity of the curve in the source regions into the target region. As shown in Figure 4, a direct connection of the boundary points in the target region cannot correctly repair the structure, and is required curve inpainting.

Figure 4. Inefficiency of line connected border pixels

These curve lines needed to complete the curves are called splines. Splines are piecewise polynomial functions that provide a suitable method for geometric modeling. The goal of using splines is interpolation, which we will explain further with an example. Suppose we have a 2D space where each point has an x and a y value. We want to find the best curve that can be drawn through these points, so that we can estimate the y value for new x values that we didn't have before. Suppose we have the y values for the known x values: x={0, 0.5, 1, 1.5, 2} (similar to the known region) and also the y values for x={5, 5.5, 6} (similar to the known region), and we need to find the y values for x={2.5, 3, 3.5, 4, 4.5} (similar to the unknown region). The goal is to obtain the correct y values for the unknown points. In Figure 5, the known points and the estimated points are shown, where the blue points are part of the known region, and the green points are the ones whose values have been estimated.

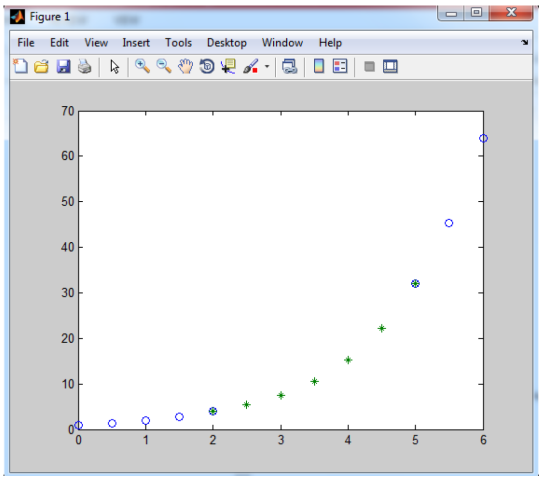

Figure 5. Estimation of missing values (green color) based on available values (blue color) with spline.

One of the notable aspects of our proposed method is that the inpainting of the sparse component of the image is highly suitable for using splines, as the sparse component contains mostly lines and curves, and the details are not very prominent. On the other hand, if splines were to be applied to the original image, they would not be able to produce efficient results. This is one of the reasons why we performed image decomposition before inpainting in this paper.

3.3. Low-Rank Component Inpainting

One of the components resulting from image analysis is the sparse component (low rank image). Considering that this component has the property of low order, we can use the concept of restoration based on order minimization. Our proposed method is based on rank minimization, in such a way that we consider a target patch, some of its pixels are in the known area and some of them are in the unknown area. For this patch, based on the known area, we find the number of n similar patches and build a tensor from the similar patches, which is shown in Figure 6.

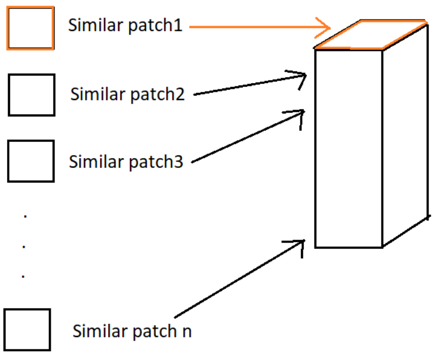

Figure 6. The process of constructing a tensor from similar patches to the target patch.

In this tensor, the target patch is also present, and some of its pixels are unknown. Then, we use one of the matrix/tensor completion methods to estimate the missing pixels in this tensor [28]. The innovation in our approach is the construction of the tensor from similar patches. In this construction, the several similar patches are migrated which make three-dimensional tensor as shown in Figure 6. Such a tensor will certainly satisfy the low-rank condition since it is constructed from similar patches.
When the matrix is low rank, the rows or columns can be obtained based on others. This coherency can be useful for filling the missing pixels from the known pixels [28]. Indeed, the missing pixels must be estimated so that the rank of image would be as low as possible. Fundamentally, tensor completion is done based on matrix completion so that the tensor decomposition is necessary in the first step and then approximation of the missing values is completed throughout the process of rank lowering. Qin applied Tensor Singular Value Decomposition (T-SVD) as a tensor decomposition and then introduced two efficient low-rank tensor approximation approaches fusing random projection techniques [28].
In our proposed method, for the low-rank component inpainting, we perform patch-by-patch filling. To estimate the missing pixels in each patch, we construct a tensor such that this tensor is built from similar patches. In general, rank minimization-based estimation methods can perform the estimation either when the target region is small or when the target region is not concentrated in one area, such as in the case of noise removal. However, our proposed method, by constructing a tensor from similar patches at each step, ensures that the number of missing pixels in a tensor is small, and the estimation of missing pixels using the rank minimization idea becomes feasible. On the other hand, if the entire image were to be considered as a tensor, and the target region was an object in the image that needed to be filled, the rank minimization-based inpainting methods would certainly not be able to produce good results.
Therefore, we first decompose the image using PCA and use the rank minimization-based inpainting algorithm only for the low-rank component, while using splines for the sparse component. Applying the rank minimization-based algorithms on the low-rank component of the image yields better results compared to applying the same algorithms on the original image. This is due to the assumption of low rank in the images, which is inherent in all the rank minimization-based inpainting algorithms, but this assumption may not hold for all images.

4. Experimental result

In this section, we have implemented our proposed method and applied it to various types of images. To demonstrate the efficiency of the proposed method, we have inpainted an image that requires restoration using different inpainting algorithms, and we have provided a sample of our experiments in Figure 7, where a single image is inpainted using different algorithms. In Figure 7, (a) shows the original image, (b) shows the mask image, (c) shows the inpainted result using the matrix approximation method [18], (d) shows the result of inpainting with the Partial Convolution algorithm [19], (e) shows the result of inpainting with the Patch Propagation algorithm [5], (f) shows the result of inpainting with the Non-Local algorithm [20], and (g) shows the image inpainted using our proposed method.

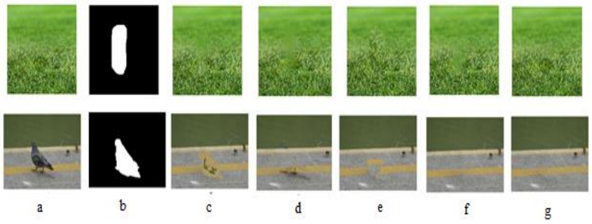

Figure 7. Comparison of the proposed method with other image inpainting methods. (a) the original image, (b) the mask image, (c) the matrix approximation method [18], (d) the Partial Convolution algorithm [19], (e) the Patch Propagation algorithm [5], (f) the Non-Local algorithm [20], (g) our proposed method.

One of the important considerations in evaluating the quality of the inpainting result is the discussion of qualitative and quantitative evaluation. In many image inpainting algorithms, qualitative evaluation (evaluation based on the human eye) is still performed. In some cases, especially when the original image is not available, there is no choice but to use qualitative evaluation to assess the final result. In some cases, similar to the images shown in Figure 7, the qualitative result clearly shows the superiority of our proposed method compared to other methods. However, in general, when the original image is available, quantitative evaluation can provide more accurate and comprehensive results for evaluating the proposed algorithm, for which the SSIM and PSNR metrics are used. For example, in Figure 8, an original image and its mask are presented, and this image has been inpainted using various methods, with the PSNR and SSIM metrics used to evaluate the performance of the different methods.

Table 1 and Table 2 provide the references for each of the methods.

In Figure 8, (a) shows the original image, (b) shows the mask image, (c) shows the inpainted result using the matrix approximation method [18], (d) shows the result of inpainting with the Partial Convolution algorithm [19], (e) shows the result of inpainting with the Patch Propagation algorithm [5], (f) shows the result of inpainting with the Non-Local algorithm [20], (g) shows HOSVD method [27], (h) shows the result of inpainting with the Patch Mixing [26], (i) shows Dimension reduction algorithm [25] and (j) shows the image inpainted using our proposed method.

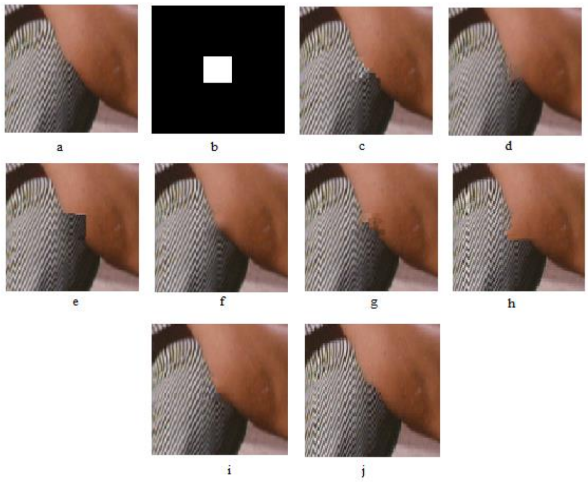

Figure 8. Comparison of the proposed method with other image inpainting algorithm. (a) the original image, (b) the mask image, (c) the result of the matrix approximation method [18], (d) the result of Partial Convolution algorithm [19], (e) the result of inpainting with the Patch Propagation algorithm [5], (f) the result of the Non-Local algorithm [20], (g) HOSVD method [27], (h) the result of inpainting with the Patch Mixing [26], (i) Dimension reduction algorithm [25] and (j) our proposed method.

Table 1 provides a quantitative comparison between different inpainting methods.

Table 1. The result of PSNR values of proposed method, compared to other algorithms

Original Image : Figure 8 (a) **Mask:** Figure 8 (b)	
Inpainting method	**PSNR value**
Patch propagation [5]	28.9274
Nonlocal inpainting [20]	36.0863
HOSVD [27]	33.7039
Patch Mixing [26]	25.7081
Approximation matrix [18]	30.1967
Partial convolution [19]	34.9558
Dimension reduction [25]	37.9773
Proposed method	38.567

Table 2. The result of SSIM values of proposed method, compared to other algorithms.

Original Image : Figure 8 (a) **Mask:** Figure 8 (b)	
Inpainting method	**PSNR value**
Patch propagation [5]	0.9527
Nonlocal inpainting [20]	0.9850
HOSVD [27]	0.9723
Patch Mixing [26]	0.9494
Approximation matrix [18]	0.9672
Partial convolution [19]	0.9794
Dimension reduction [25]	0.9876
Proposed method	0.992

The reality is that one of the most important factors that affects the efficiency of an image inpainting algorithm is the complexity of the image that the algorithm is supposed to inpaint, or in other words, the nature of the information present in the image.

Along with that, the size of the target region can also affect the efficiency of an inpainting method, such that the larger the target region, the more difficult the image inpainting becomes, and the efficiency of the inpainting algorithm decreases. The truth is that we can have different target regions, and for each of them, the ratio of the size of the target region to the known region is important. For target regions that are scattered throughout the image, such as text, noise, or blocks, inpainting is simpler compared to the case where we need to remove an object or a concentrated region in the image.

It is important to note that we cannot provide an exact size or ratio for the target region compared to the known region, because in addition to the size of the target region, the efficiency of the algorithm largely depends on the nature of the image information. For example, if the image information lacks complex geometric structures and has a simple, repeating texture pattern, the efficiency of the algorithm will be high even with an increase in the size of the target region, as the inpainting of such an image is essentially simple.

However, if the image contains complex geometric structures and different textures in different regions are observed, the image inpainting will be a more difficult task. In Figure 9, an image can be observed where a large part of it is made up of a uniform texture, and it is to be inpainted with three different target region sizes.

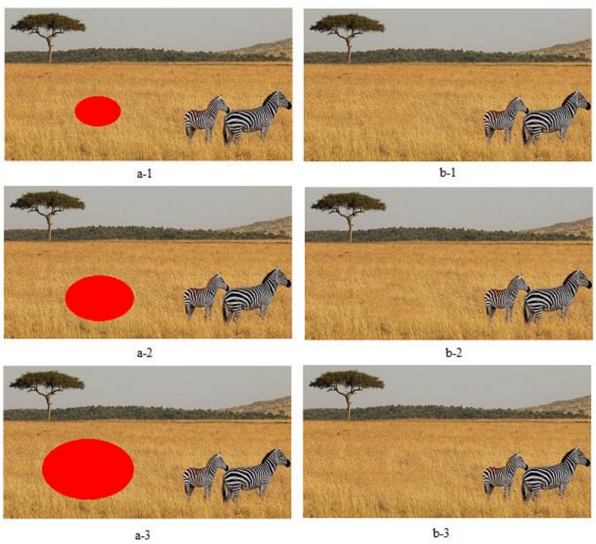

Figure 9. The effect of target area size for inpainting in the simple texture region. (a-1, a-2, a-3) are original images with the target regions, (b-1, b-2, b-3) are the inpainted results using our proposed method.

As can be observed, in all three images, satisfactory results are obtained after inpainting. On the other hand, if the image contains complex geometric structures, increasing the size of the target region does not yield satisfactory results. In other words, as the size of the target region in an image containing geometric structures increases, the efficiency of the inpainting algorithm decreases. An example of such an image can be observed in Figure 10.

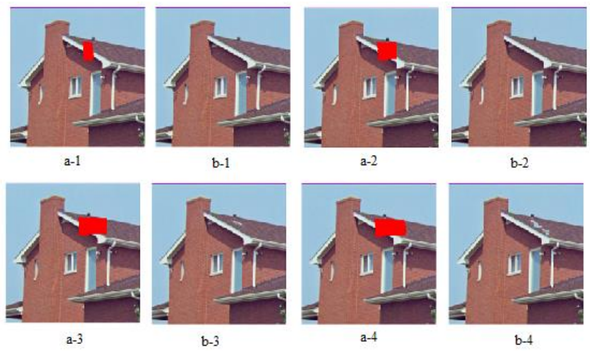

Figure 10. The effect of target area size for inpainting in the complex region. (a-1, a-2, a-3, a-4) are original images with the target regions, (b-1, b-2, b- 3, b-4) are the inpainted results using our proposed method.

In Figure 10, an image containing complex geometric structures can be seen, with four different target region sizes. As the target region size increases, the quality of the result decreases, and artifacts are introduced after inpainting.

5. Discussion

Our proposed method has both strengths and weaknesses. First, we will discuss the weaknesses of the proposed method. One of the weaknesses of the proposed method is the issue of the size of the target region. Since in our proposed method we have used rank minimization and splines for inpainting, the target region should not be too large. If the target region is very large compared to the size of the image, our proposed algorithm may not be able to produce satisfactory results.

Another important issue in the discussion of image inpainting is the required execution time for inpainting. The reality is that image inpainting algorithms that operate in two stages have a lower speed and are slower compared to methods that perform inpainting in a single stage. However, they can produce results with higher quality. The importance of quality in the field of image inpainting is greater than the speed of execution, and the majority of image inpainting methods have focused on improving quality. In fact, there is always a trade-off between the speed of algorithm execution and the quality of inpainting. Our proposed method is a two-stage process, meaning it requires two estimations of the missing pixels, and therefore may require more time. However, the increase in time is not such that the execution time becomes unacceptable. Our approach in this paper is offline image inpainting, so that we can create a desirable quality after inpainting. Therefore, the issue of quality is the main priority for us. As you can see in Table 3, we have performed a comparative analysis on the execution time of our proposed algorithm with other prominent algorithms to examine this issue. It is noticeable that several images are considered for the comparative analysis on the execution time, which can be seen in Figure 11.

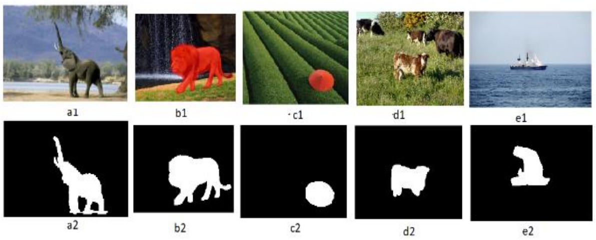

Figure 11. several images considered for the comparative analysis on the execution time, (a1)(b1)(c1)(d1)(e1) are the original images and (a2)(b2)(c2)(d2)(e2) are the relates masks.

Table 3. The result of execution time in our proposed algorithm with Matrix approximation [18] and Patch propagation methods [5].

Method	**Original image and mask in figure number**				
	Image Mask	Image Mask	Image Mask	Image Mask	Image Mask
	Fig.11 (a1) (a2)	Fig.11 (b1) (b2)	Fig.11 (c1) (c2)	Fig.11 (d1) (d2)	Fig.11 (e1) (e2)
Patch Propagation [5]	92.93	294.26	33.16	144.29	127.14
Matrix Approximation [18]	33.21	152.47	6.77	97.04	52.51
Our method	127.46	450.74	41.01	242.87	181.06

Our algorithm has an important strength, which is the ability to inpaint curves, lines, and generally the ability to inpaint lost edges in the target region. In images where the target region is part of a curve or a line, inpainting methods cannot perform good inpainting, and the image after inpainting has artifacts. However, in our proposed algorithm, due to image decomposition and the use of splines on the sparse component of the image, curves and lines can be well recovered, while the low-rank component of the image is also inpainted in parallel by a low-rank minimization algorithm.

6. Conclusion

In this paper, a two-stage method for image inpainting is proposed that uses the idea of splines and low-rank minimization for inpainting. In our method, there is a need to decompose the image and find its components, for which we have used PCA. After applying PCA and obtaining the image components, we get a sparse component and a low-rank component. In this paper, we independently inpaint each component, and finally, we obtain the original image by combining the two inpainted components. This helps us achieve good inpainting even in images where the target region has many edges and complexities. Another important point is the extraction of the low rank component and construction the tensor from the similar patches, which helps improve the efficiency of the low-rank minimization-based algorithm. Because many default images do not have low rank, and therefore, applying this algorithm does not produce good results. Finally, we have compared our proposed method with other inpainting methods, and the results indicate the superiority of our algorithm compared to other methods.

Acknowledgement

This research is funded by Semnan University, research grant 20665.

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یک الگوریتم ترکیبی ترمیم تصویر : ترکیب کمینه کردن مرتبه و روش Spline

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چکیده:

ترمیم تصویر یکی از موضوعات مورد توجه در حوزه پردازش تصویر محسوب می‌شود به طوری‌که تا کنون روش‌های مختلفی در این حوزه ارائه داده شده است. . اما این مساله هنوز هم با چالش‌های متعددی روبرو می‌باشد. برای نمونه ممکن است یک الگوریتم ترمیم برای دسته خاصی از تصاویر خوب عمل نماید و برای سایر تصاویر عملکرد ضعیفی داشته باشد. ما در این مقاله تلاش می‌کنیم به کمک PCA تصویر را به دو جزء مرتبه پایین و تُنُک تجزیه کرده و هر جزء را به صورت مستقل ترمیم نماییم به طوری‌که در ترمیم جزء مرتبه پایین از یک الگوریتم مبتنی بر کمینه کردن مرتبه استفاده می‌کنیم و در ترمیم جزء تُنُک، از مفهوم spline اســتفاده می‌نماییم. به کمک spline می‌توانیم لبه‌ها و خطوط را به خوبی ترمیم نماییم در حالی که ترمیم این نواحی در اکثر الگوریتم‌ها با چالش مواجه است. همچنین در ترمیم جز مرتبه پایین، در هر مرحله یک تنسور می‌سازیم و پیکسل‌های ناموجود در تنسور را تقریب می‌زنیم در نتیجه کارایی ایده کمینه کردن مرتبه در ترمیم تصــویر را تا حد زیادی بالا می‌بریم. در نهایت روش پیشــنهادی خود را در ترمیم انواع مختلف تصاویر به کار گرفته‌ایم و بر اساس معیارهای PSNR و SSIM روش خود را ارزیابی نموده‌ایم که نتایج به دست آمده نشان از کارایی روش پیشنهادی ما نسبت به سایر روش‌های ترمیم دارد.

کلمات کلیدی: ناحیه هدف، spline، کمینه کردن مرتبه، تنسور.