Journal of Artificial Intelligence and Data Mining (JAIDM), Vol. 10, No. 3, 2022, 433-447.

Shahrood University of Technology

Journal of Artificial Intelligence and Data Mining (JAIDM)
Journal homepage: http://jad.shahroodut.ac.ir

Research paper

A random Scheme to Implement *m*-Connected *k*-Covering Wireless Sensor Networks

Vahid Ghasemi[1] and Ali Ghanbari Sorkhi[2*]

[1]*Department of Computer Engineering, Faculty of Information Technology, Kermanshah University of Technology, Kermanshah, Iran*
[2]*Department of Computer Engineering, University of Science and Technology of Mazandaran, Behshahr, Iran.*

Article Info

Article History:
Received 22 February 2022
Revised 24 April 2022
Accepted 08 May 2022

DOI:10.22044/jadm.2022.11695.2319

Keywords:
M-connectivity, K-coverage, Wireless Sensor Networks, Support Sets.

**Corresponding author: ali.ghanbari@mazust.ac.ir (A. Ghanbari Sorkhi).*

Abstract

Deploying the *m*-connected *k*-covering (MK) wireless sensor networks (WSNs) is crucial for a reliable packet delivery and target coverage. This paper proposes implementing random MK WSNs based on the expected *m*-connected *k*-covering (EMK) WSNs. We define EMK WSNs as random WSNs mathematically, expected to be both *m*-connected and *k*-covering. Deploying random EMK WSNs is conducted by deriving a relationship between *m*-connectivity and *k*-coverage, together with a lower bound for the required number of nodes. It is shown that EMK WSNs tend to be MK asymptotically. A polynomial worst-case and linear average-case complexity algorithm is presented in order to turn an EMK WSN into MK in non-asymptotic conditions. The *m*-connectivity is founded on the concept of support sets to strictly guarantee the existence of m disjoint paths between every node and the sink. The theoretical results are assessed via the experiments, and several metaheuristic solutions are benchmarked to reveal the appropriate size of the generated MK WSNs.

1. Introduction

The advent of technologies such as the Internet of things [1], pervasive computing [2], smart environments/cities [3], e-healthcare [4], and surveillance systems [5] has multiplied the importance of a proper deployment of Wireless sensor networks (WSNs) in the recent years. WSNs are the cornerstone of data acquisition in such high-end contemporary applications. A WSN consists of a bunch of communicating motes, sensing an area of interest (AoI). The sensors are required to cover the areas, barriers or individual targets in AoI [6]. They collect a broad range of data types: temperature, humidity, pressure, vibration, sound, biomedical information, etc.

Generally, WSNs can be deployed by either a pre-planned or a stochastic strategy [7, 8]. However, preserving connectivity among the sensor nodes and target coverage are two challenging facets in both scenarios. It is due to the fact that the sensor nodes have limited resources (such as communication range and power supplies), and are also susceptible to failure by external events. Therefore, there may be link outages (which in turn corrupt routing packets toward the sink) and also target coverage loss in AoI. *M*-connected *k*-covering (MK) WSNs are deployed to cope with these problems. In a *k*-covering WSN, every target point/area is covered by at least k sensor nodes. Also a WSN is called *m*-connected if there exist at least m disjoint paths between each pair of nodes. The definition of *m*-connectivity can be reduced to the existence of m disjoint paths between each node and the sink. Having selected proper values for m and k, MK WSNs provide a reliable packet delivery (via multi-hop routing) and target coverage. It is typically struggled to minimize the number of nodes in an MK WSN to lower the deployment costs. However, deploying an MK WSN with a minimum number of nodes is an NP-complete problem [9-11].

Recently, a number of works have grappled with solving the MK WSN problem via metaheuristic algorithms [12, 13]. These methods generate a random dense primary potential network in order to ensure a primitive MK WSN from scratch. Then the potential nodes are pruned via a metaheuristic minimization process such that the MK constraint is preserved. Notwithstanding these methods are dramatically prone to local minima due to the network size, the question of: “how much dense an initial random WSN should be, to expect both m-connectivity and k-coverage properties” has been ignored. On one hand, if the size of the initial random network is too small, the MK constraint may be violated; on the other hand, a dense network infuses a great number of optimization variables (i.e. associated with the network size) into the meta-heuristic optimization process, increasing the chance of suboptimal local minima solutions. In fact, an overlooked crux in the presented metaheuristic approaches has been to generate a primary random MK WSN of a proper size and avoid overestimations. Although the study of m-connectivity and k-coverage in random WSNs has been conducted in some works, a relaxed hypothesis for m-connectivity has been applied [8]. In such works, a WSN is typically considered m-connected if the degree of every node is greater than or equal to m. This hypothesis can violate m-connectivity literally. Figure 1 illustrates a violating scenario for m = 2. In this figure, the neighboring nodes in a WSN are connected by solid line edges. As it can be seen, the degree of every node is greater than or equal to 2. However, no 2 disjoint paths exist from nodes at either regions A and B (inside the dashed borders) to the sink because all paths from the nodes at these regions to the sink pass through the common node 18. Therefore, the nodes at regions A and B are not 2 connected despite the degrees of all nodes are greater than or equal to 2. This paper is motivated by the above question, under the strict definition of m-connectivity. The answer can yield a fair-sized random MK WSN.

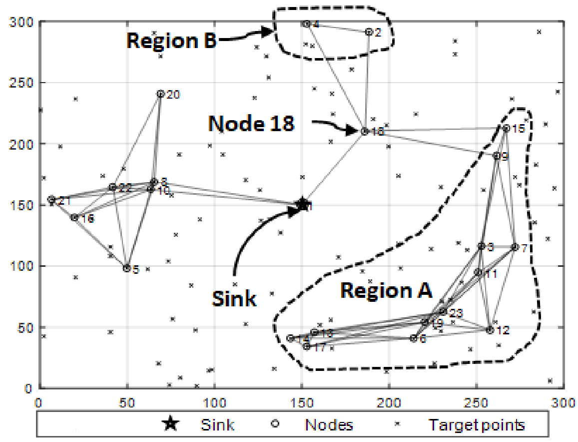

Figure 1. A random WSN that is not 2-connected, despite the degree of every node is greater than or equal to 2.

A novel heuristic is proposed in this paper to cope with the problem of size overestimation in random MK WSN. The idea is to generate a random MK WSN whose size is approximated based on the mathematical expectations of m-connectivity and k-coverage. For this purpose, we introduce *expected m-connected k-covering (EMK) WSNs*, which are random WSNs mathematically expected to be both k-covering and m-connected. This way, a lower bound on the size of a random EMK WSN is calculated as a function of m, k, and some network characteristics, a priory. We show that an EMK WSN tends to be MK in asymptotic conditions (i.e. for large values of m and k). An EMK WSN can be turned into an MK by adding a limited number of nodes, with polynomial worst-case and linear average-case complexities, as will be discussed. Therefore, to create a random MK WSN, it is proposed to generate an interim EMK WSN with a pre-determined size based on m and k, and then reform it to become MK. Also the m-connectivity is studied under the strict condition of existence of m disjoint paths from every node to the sink based on the concept of *support sets*. This way, m-connectivity can be guaranteed. Several metaheuristic solutions have been benchmarked to reveal the proper size of the generated MK WSNs. The main contributions of the present work can be summarized as:

- EMK WSNs are introduced for deploying random MK WSNs.
- The m-connectivity is defined based on the concept of support sets to guarantee the existence of m disjoint paths from every node to the sink.
- The size of the expected m-connected, the expected k-covering, and the EMK WSNs have been analyzed mathematically and then assessed via simulations.
- A probabilistic algorithm with polynomial worst-case and linear average-case complexities is delivered to turn an EMK WSN into an MK WSN.
- A relationship between m-connectivity and k-coverage is presented analytically and assessed via simulations.
- Some metaheuristic approaches for solving the minimum MK WSN problem are benchmarked to reveal the suitable size of the random MK WSNs generated based on the EMK structure.

The remainder of the paper is organized as what follows. Some related works are reviewed in Section 2. Some preliminary definitions and the network model are taken in Section 3. We define support sets in Section 4, and prove their contribution in *m*-connected WSNs. Sections 6 and 7 analyze the expectations of support and cover set sizes, respectively. Section 8 discusses the relation between *m*-connectivity and *k*-coverage and how to generate EMK WSNs. Section 9 presents a probabilistic algorithm to reform an EMK WSN into MK. The complexity of this algorithm is calculated in Section 10. Section 11 delivers the experimental results, and finally, Section 12 concludes the paper.

2. Related Works

The low-cost development of MK WSNs is an np-hard problem [9-11]. Therefore, an approximate solution is required. The problem is generally solved via heuristic and metaheuristic approaches. The cost is directly associated with the number of nodes. Therefore, it is always struggled to minimize the number of nodes. Some heuristic methods solve this problem by scattering a number of sensor nodes based on pre-defined regular topologies (e.g. triangular, rectangular, hexagonal, etc.) [7, 14]. Along with the regular topology-based solutions, metaheuristic algorithms solve this problem by generating and optimizing random topologies. Some recent metaheuristic approaches are as follow:

In [6], a scheme based on biogeography-based optimization (BBO) is used in order to solve the problem. The proposed method provides an encoding for the habitat representation, and formulates an objective function along with the BBO's operators.

In [15, 16], a network of potential positions for placing sensor nodes is pre-specified in a grid or a random style. Then a scheme based on the imperialist competitive algorithm (ICA) is proposed to solve the MK WSN problem by selecting a subset of initial potential sensor positions for node placement. Also [16] improves the ICA method by enveloping the possibility of immigration for colonies from weak to stronger empires into the standard ICA. This method is called immigrant ICA (IICA).

In [17], an optimization approach based on a hybrid *tunicate swarm optimizer (TSO)* and *salp swarm optimizer (SSO)* is proposed in order to solve the minimum MK WSN problem. In this approach, a potential initial WSN is generated and pruned to yield a final MK WSN. In [18] and [19], two schemes based on *genetic algorithm* (GA) and *gravitational search algorithm* (GSA) have been proposed, respectively. These methods are identically based on generating the initial potential WSNs and minimizing the same objective function as in [17] to yield an optimal MK WSN. *Particle swarm optimization (PSO)* and *differential evolution (DE)* have been other instances of metaheuristic evolutionary algorithms for solving the minimum MK WSN problem in the same way [12].

In [20], a mathematical model called Nelder-Mead method is applied in the shuffled frog leaping algorithm to improve the local search for solving the minimum MK WSN problem. Like the previous approaches, the idea has been to generate a potential network and prune the potential positions by optimizing a specific objective function.

In addition to the previous approaches, some fault tolerant WSN schemes opt the same strategy to obtain the initial MK WSNs [21, 22]. They restore connectivity or coverage status by relocating the nodes in the sense of failures. All of the above-mentioned schemes generate primitive random potential WSNs, which are taken *m*-connected and *k*-covering for granted. Therefore, studying the *m*-connectivity and *k*-coverage properties in random WSN is of crucial importance. This issue has been studied in some other works as follows.

In [8], *m*-connectivity and *k*-coverage problem for uniformly deployed 3D AoIs has been studied. In this work, the sensors are heterogeneous in terms of the sensing range, communication range, and the possibility of being alive. Similar to this work, [23] analyzes the coverage and connectivity in homogenous directional 2D WSNs, considering the in- and out-degree of nodes. In [24] and [25], the critical density of sensor nodes has been calculated to achieve both network connectivity phase transition (NCPT) and sensing-coverage phase transition (SCPT). Both of these works focus on 1-coverage and 1-connectivity.

In [26], the critical sensor density (CSD) for the desired coverage ratio is calculated. In this work, the sensors are uniformly deployed in a 2D convex polygon-shaped AoI. The approaches of [27] and [28] are two other instances that study coverage in WSNs. The first studies the coverage in bounded areas, while the latter exploits the fundamental limits of coverage based on the stochastic data fusion models that fuse noisy measurements of multiple sensors. *M*-connectivity is not considered in the above-mentioned works.

When it comes to the *m*-connectivity analysis, the characteristic is typically attributed to the number of neighboring nodes in most related works including those mentioned above. It can violate *m*-connectivity, as shown in Figure 1. In addition, the metaheuristic methods manage to solve the minimum EMK problem without assessing the *m*-

connectivity and k-coverage properties in the primary random potential WSNs. This work analyzes both properties under the strict condition of *m*-connectivity, and proposes a method for generating random MK WSNs to plug the gap.

3. Primary Definitions and Network Model

WSN is considered as a set of uniformly scattered sensor nodes, namely V, and a sink node S, in a given 2D AoI. The sensing and communication ranges of all sensor nodes are considered static, and denoted by R_s and R_c, respectively. The sensors are considered omnidirectional with a binary communication/sensing model. The target point coverage is considered for the purpose of this paper, i.e. a set of targets, namely T, are uniformly distributed in AoI. General notations are summarized in Table 1, and some preliminary definitions are taken in the following.

Table 1. General notations.

Notation	Description
A	Area of AoI
S	Sink node
T	Set of targets in WSN
V	Set of sensor nodes in WSN
R_s	Sensing rang.
R_c	Communication range
$N(ב)$	Set of neighbors of node ב
$sup(ב)$	Support set of node/set of nodes ב
$cov(ב)$	Cover set of target ב
$\|ב-\vartheta\|_2$	Euclidian distance between nodes/targets ב and ϑ
$\|ב\|$	Size of a set ב
n	Number of nodes
$N^h(ב)$	Set of neighbors at a maximum h-hop distance from node ב
$v_i \rightarrow v_j$	A path sequence from node v_i to v_j
<.>	An ordered sequence of nodes
$E(.)$	Mathematical expectation
$\mathbb{1}_\alpha(\beta)$	Indicator function
A_v^{sup} or $A_{x,y}^{sup}$	Support area of a node v or a node located at (x,y)
$A_{x,y}^{cov}$	Coverage area of a target, located at (x,y)

Definition 2 (cover set). The cover set of a target $\boldsymbol{t}$ is denoted by $\boldsymbol{cov(t)}$, and defined as:

$$\mathrm{cov}(t)=\left\{x\in V \,\middle|\, \|x-t\|_2\le R_s\right\}. \quad (1)$$

$cov(t)$ includes the nodes that cover target t. The geometrical location of such nodes, known as the *coverage area* of t, is a disk of radius R_s, centered at the target.

Definition 3 ($\boldsymbol{k}$-coverage). A target $\boldsymbol{t}$ is *k*-covered if $|\boldsymbol{cov(t)}|\ge \boldsymbol{k}$, and a WSN is called *k*-covering if all its targets are *k*-covered.

Definition 4 ($\boldsymbol{m}$-connectivity). A node $\boldsymbol{v}\notin \boldsymbol{N(S)}$ is *m*-connected if there exist at least $\boldsymbol{m}$ disjoint paths from $\boldsymbol{v}$ to the sink $\boldsymbol{S}$, and a WSN is *m*-connected if every node $\boldsymbol{v}\notin \boldsymbol{N(S)}$ is *m*-connected.

Definition 5 (MK WSN). A WSN is called *m*-connected *k*-covering (MK) if it is both *m*-connected and *k*-covering.

Definition 6 (expected $\boldsymbol{k}$-covering WSN). A random WSN is called expected *k*-covering if $\boldsymbol{E}(|\boldsymbol{cov(t)}|)\ge \boldsymbol{k}$, for an arbitrary target $\boldsymbol{t}$, where $\boldsymbol{E}(.)$ is the mathematical expectation.

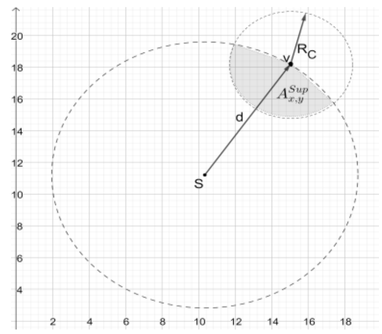

Figure 2. Shaded area (i.e. $A_{x,y}^{Sup}$) is geometrical bounds of support area of v.

4. Support Sets

The *support set* of a node includes the neighbor nodes closer to the sink than itself. These nodes are geometrically placed in the intersection area of the disk centered at S with radius $d=\|S-v\|_2$ and the communication area of v. This region, called *the support area* of v, is depicted in Figure 2. The support sets are defined as:

Definition 7 (support set). A support set is denoted by $\boldsymbol{sup(v)}$ for every node $\boldsymbol{v}\notin \boldsymbol{N(S)}$, and defined as:

$$\sup(v)=\left\{x\in N(v) \,\middle|\, \|x-S\|_2\le \|v-s\|_2\right\}. \quad (2)$$

Also the support set of a set of sensor nodes V' is defined as the union of support sets of all nodes in V', as:

$$\sup(V')=\bigcup_{v\in V'}\sup(v). \quad (3)$$

The support sets are crucial since their size can guarantee *m*-connectivity in a WSN. The following lemma addresses this issue.

Lemma 1. A WSN is *m*-connected if $\forall \boldsymbol{v}\notin \boldsymbol{N(S)}\ |\boldsymbol{sup(v)}|\ge \boldsymbol{m}$.

Proof. Order the sink's non-neighboring nodes based on their distance to it, non-decreasingly. Let denote this ordered sequence by $Q_z=<v_1,\dots,v_z>$, where $\forall i<j\ \|v_i-\mathrm{S}\|_2\le \|\mathrm{v_j}-\mathrm{S}\|_2$. The proof is conducted by induction on z. Let $z=1$ as the base case. There exist at least m disjoint paths from v_1 to the sink because $|\sup(v_1)|\ge m$ and $sup(v_1)\subseteq N(S)\cap N(v_1)$. Now, assume the induction hypothesis that the

nodes of Q_z are m-connected and prove for $Q_{z+1} =< Q_z, v_{z+1} >$, where $z > 1$.
Based on the Menger's theorem [29], a node v_i is m-connected if the size of the minimum vertex cut for v_i and the sink equals m (i.e. at least m nodes must be omitted to disconnect v_i from the sink). Therefore, to prove m-connectivity for the nodes in $Q_{z+1,}$ it is sufficient to prove that the minimum size of vertex cuts for each node in Q_{z+1} and the sink equals m. For this purpose, since the nodes of Q_z are m-connected, the minimum size of their vertex cuts equals m. Also if node v_{z+1} is added to Q_z, it will not reduce the minimum vertex cut size because it is connected to at least m supporting neighbors in Q_z (i.e. $|\mathrm{sup}(v_{z+1})| \geq m$ and $\mathrm{sup}(v_{z+1}) \subseteq Q_z$). Thus the minimum vertex cut size for nodes in Q_{z+1} and the sink equals m and the proof is complete. ■

Inspired by lemma 1, we define an *expected m-connected WSN* based on the support sets as follows. Such networks are mathematically expected to be m-connected.

Definition 8 (expected *m*-connected WSN). A random WSN is called expected *m*-connected if $\boldsymbol{E}(|\,\boldsymbol{sup}(\boldsymbol{v})\,|) \geq \boldsymbol{m}$ for an arbitrary node $\boldsymbol{v} \notin \boldsymbol{N}(\boldsymbol{S})$.

Similarly, an *expected m-connected k-covering (EMK)* WSN is defined as follows:

Definition 9 (EMK WSN). A random WSN is called expected *m*-connected *k*-covering (EMK) if it is both expected *m*-connected and expected *k*-covering.

A random EMK WSN can yield an appropriate initialization for generating a random MK WSN. It is crucial since it will be asymptotically *m*-connected and *k*-covering with a probability tending to 1, as will be discussed.

5. Problem Statement

Random EMK WSNs can be regarded as approximations of MK WSNs. Such networks are *expected* to be both *m*-connected and *k*-covering mathematically. The problem of deploying random MK WSNs is defined based on EMK WSNs in this paper.

Problem statement- Given an AoI with a set of random targets T, generate a random MK WSN by reforming an EMK WSN.

An EMK WSN is required in advance to solve the problem mentioned above. The following sections provide the essential analysis for creating random EMK WSNs. We deliver a probabilistic algorithm to turn an EMK WSN into an MK, with *polynomial worst-case* and *linear average-case* complexities (section 10). The border effect is not considered in the calculations of the forthcoming sections.

6. Expectation of Support Set Size in Random WSNs

Consider the support area of an arbitrary node $v \notin N(s)$ located at (x, y). The area of this region, denoted by $A_{x,y}^{Sup}$ (Figure 2), depends on the coordinates of v. Since the nodes are scattered uniformly in the AoI, for a node $v \notin N(s)$, located at (x, y), the probability that an arbitrary node ω falls within $\mathrm{sup}(v)$ will be:

$$p_{x,y}\left(\omega \in \mathrm{sup}\left(v\right)\right) = \frac{A_{x,y}^{Sup}}{A}. \tag{4}$$

$A_{x,y}^{Sup}$ can be generally calculated as:

$$A_{x,y}^{Sup} = d^2 \arccos\left(1 - \frac{R_c^2}{2d^2}\right) + R_c^2 \arccos\left(\frac{R_c}{2d}\right) - d\sqrt{R_c^2 - \frac{R_c^4}{4d^{2'}}} \tag{5}$$

where $d = \sqrt{x^2 + y^2}$. $A_{x,y}^{sup}$ is highly dependent on the distance of node v from the sink, i.e. parameter d. Albeit, a lower bound for $p_{x,y}(\omega \in \mathrm{sup}(v))$ can be calculated. The more v gets closer to the sink, the less $A_{x,y}^{Sup}$ will be. Thus $A_{x,y}^{Sup}$ is minimized when v is at the closest possible distance, i.e. $d = R_c$, from the sink. Replacing $d = R_c$ in (5) and then (4) yields:

$$p_{x,y}\left(\omega \in \mathrm{sup}\left(v\right)\right) \geq \frac{\alpha R_c^2}{A}, \tag{6}$$

where $\alpha = (\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$.

Since $\mathbb{1}_{\mathrm{sup}(v)}(\omega)$ can be regarded as a Bernoulli variable with parameter $p_{x,y}(\omega \in \mathrm{sup}(v))$, the size of $sup(v)$, i.e. $|\mathrm{sup}(v)| = \sum_{\omega \in V} \mathbb{1}_{\mathrm{sup}(v)}(\omega)$, will be a binomial variable with distribution:

$p_{x,y}(|\mathrm{sup}(v)| = m) =$

$$p_{x,y}\left(\left|\mathrm{sup}\left(v\right)\right| = m\right) = \binom{n-1}{m}.p_{x,y}\left(\omega \in \mathrm{sup}\left(v\right)\right)^m. \left(1 - p_{x,y}\left(\omega \in \mathrm{sup}\left(v\right)\right)\right)^{n-m-1}. \tag{7}$$

Therefore, the expectation of $|\mathrm{sup}(v)|$, given that node $v \notin N(S)$ is placed at (x, y) will be:

$$E_{x,y}\left(\left|\mathrm{sup}\left(v\right)\right|\right) = \left(n-1\right) p_{x,y} \left(\omega \in \mathrm{sup}\left(v\right)\right). \tag{8}$$

Replacing (6) in (8) yields:

$$E_{x,y}\left(\left|\mathrm{sup}\left(v\right)\right|\right) \geq \left(n-1\right) \frac{\alpha R_c^2}{A}. \tag{9}$$

The expected size of the support set of an arbitrary node v, independent from its position, can be calculated as:

$$E\left(\left|\sup(v)\right|\right) = \iint_{A'} \frac{1}{A} E_{x,y}\left(\left|\sup(v)\right|\right) d(x) d(y), \quad (10)$$

where $A' = A - \pi R_c^2$. Applying (9) in (10) yields:

$$E\left(\left|\sup(v)\right|\right) \geq \iint_{A'} \frac{(n-1)\alpha R_c^2}{A} d(x) d(y) = \frac{\alpha(n-1) R_c^2 A'}{A^2} \quad (11)$$

The following proposition results directly from (11) by solving $m = \frac{\alpha(n-1)R_c^2 A'}{A^2}$ for n.

Result 1. A minimum number of $n = \left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1$ uniformly scattered nodes is required to achieve an expected *m*-connected WSN.

Therefore, to generate an expected *m*-connected WSN, it is sufficient to uniformly scatter a minimum number of $n = \left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1$ nodes based on result 1 in AoI.

It should be noted that given an AoI, when $m \to \infty$, an expected *m*-connected WSN will be *m*-connected with a probability tending to 1. The weak law of large numbers (LLN) can justify this issue, i.e. since $|sup(v)|$ is a binomial variable, the relation $P(\left||\mathrm{sup(v)}| - E(|\mathrm{sup(v)}|)\right| \geq \epsilon) \to 0$ holds as $m \to \infty$, according to the weak LLN (note that $n \to \infty$ as $m \to \infty$). Therefore, the support set size of an arbitrary node v approaches $E(|\mathrm{sup(v)}|)$ with a probability tending to 1, i.e. $\mathrm{plim}_{m\to\infty}|sup(v)| = E(|sup(v)|)$. As a result, since $E(|\mathrm{sup(v)}|) \geq m$, we will have $|\mathrm{sup(v)}| \geq m$ with a probability tending to 1, asymptotically. This issue is assessed experimentally and stated by the following proposition.

Result 2. Given an AoI, R_c, and R_s, an expected *m*-connected WSN will be *m*-connected with a probability tending to 1 as $m \to \infty$.

7. Expectation of a Cover Set Size in Random WSNs

In order to calculate the expectation of the cover set size, consider an arbitrary target $t \in T$, located at (x, y). We have:

$$p\left(\omega \in \mathrm{cov}(t)\right) = \frac{A_{x,y}^{\mathrm{cov}}}{A}. \quad (12)$$

Similar to the support sets, $|cov(t)|$ is also a binomial variable with distribution:

$$p\left(\left|\mathrm{cov}(t)\right| = k\right) = \binom{n}{k} . p\left(\omega \in \mathrm{cov}(t)\right)^k . \left(1 - p\left(\omega \in \mathrm{cov}(t)\right)\right)^{n-k} \quad (13)$$

Thus the expectation of $|\mathrm{cov}(t)|$ will be:

$$E_{x,y}\left(\left|\mathrm{cov}(t)\right|\right) = np\left(\omega \in \mathrm{cov}(t)\right). \quad (14)$$

Replacing $p(\omega \in cov(t))$ from (12) in (14) yields:

$$E_{x,y}\left(\left|\mathrm{cov}(t)\right|\right) = \frac{nA_{x,y}^{\mathrm{cov}}}{A}. \quad (15)$$

The expected size of the cover set of an arbitrary target t, independent from its position, can be calculated as:

$$E\left(\left|\mathrm{cov}(t)\right|\right) = \iint_{A'} \frac{1}{A} E_{x,y}\left(\left|\mathrm{cov}(t)\right|\right) d(x) d(y). \quad (16)$$

$A_{x,y}^{cov}$ is dependent on the position of the target. If a target is located at a distance less than R_s from the brim, then a portion of its coverage area falls outside AoI. The more t gets closer to the brim, the less its coverage area will be. The minimum coverage area, denoted by A_{min}^{cov}, is achieved when a target is placed at the brim. Applying $A_{x,y}^{cov} \geq A_{cov}^{min}$ in (15) and then (16) yields:

$$E\left(\left|\mathrm{cov}(t)\right|\right) \geq \iint_{A} \frac{nA_{\mathrm{min}}^{\mathrm{cov}}}{A^2} d(x) d(y) = \frac{nA_{\mathrm{min}}^{\mathrm{cov}}}{A}. \quad (17)$$

If the border effect is ignored, then we have $A_{min}^{cov} = \pi R_s^2$. The following proposition results directly from (17) by solving $k = \frac{nA_{min}^{cov}}{A}$ for n.

Result 3. A minimum number of $\left\lceil \frac{kA}{A_{min}^{cov}} \right\rceil$ uniformly scattered nodes are required to achieve an expected *k*-covering WSN.

Therefore, in order to generate an expected *k*-covering WSN, it is sufficient to scatter a minimum number of uniformly $\left\lceil \frac{kA}{A_{min}^{cov}} \right\rceil$ nodes based on result 3 in AoI.

Similar to the expected *m*-connectivity, the expected *k*-covering WSNs will be *k*-covering with a probability tending to 1, asymptotically. Since $|cov(t)|$ is a binomial variable for a random target t, we have $P(\left||\mathrm{cov(t)}| - E(|\mathrm{cov(t)}|)\right| \geq \epsilon) \to 0$ as $k \to \infty$, according to the weak LLN (note that $n \to \infty$ as $k \to \infty$). Therefore, the size of the cover sets approaches to $E(|\mathrm{cov(t)}|)$ with a probability tending to 1, i.e. $\mathrm{plim}_{k\to\infty}|\mathrm{cov(t)}| =$

$E(|\text{cov(t)}|)$. As a result, since $E(|\text{cov(t)}|) \geq k$, we will have $|\text{cov(t)}| \geq k$ for a target t asymptotically. Thus the following result ensues. It is assessed experimentally too.

Result 4. Given an AoI, R_c, and R_s, an expected k-covering WSN will be k-covering with a probability tending to 1 as $k \to \infty$.

8. Generating an EMK WSN

In order to generate EMK WSNs, one must first study the relationship between m-connectivity and k-coverage. The following lemma addresses this issue.

Lemma 2. In a uniformly deployed WSN, if the WSN is expected m-connected, then it is expected k-covering with $\boldsymbol{k} = \left\lfloor \boldsymbol{2m}\left(\frac{R_s}{R_c}\right)^{\boldsymbol{2}} \right\rfloor$.

Proof. Lets A_v^{sup} and B_v^{sup} denote the support area of an arbitrary node v and its mirror reflection across line l, as depicted in Figure 3. Both A_v^{sup} and B_v^{sup} are sub-areas of the communication area of v.

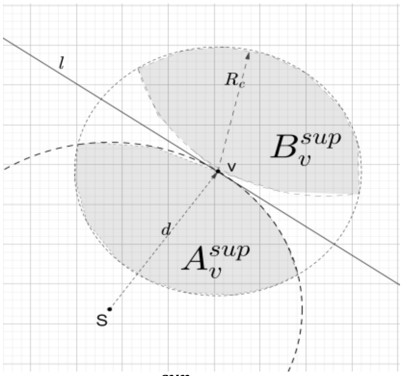

Figure 3. Support area A_v^{sup} mirror reflected across line l, yielding B_v^{sup} in communication area of v.

Lets n_{sup}, n'_{sup}, and n_c denote the number of nodes in A_v^{sup}, B_v^{sup}, and the communication area of v, respectively. Since $|B_v^{sup}| = |A_v^{sup}|$, the expected number of nodes in B_v^{sup} and A_v^{sup} are identical, and greater than or equal to m, i.e. $E(n_v) = E(n'_v) \geq m$ since the WSN is expected m-connected. Also since A_v^{sup} and B_v^{sup} are sub-areas of the communication area, we have:

$$E(n_c) \geq E(n_{\text{sup}}) + E(n'_{\text{sup}}) \geq 2m. \quad (18)$$

Since the nodes and the targets are uniformly scattered, the density of nodes will be $\frac{E(n_c)}{\pi R_c^2}$; therefore, the expectation of a target's cover set size will be proportional to the relationship between sensing and communication areas, as:

$$E(|\text{cov}(t)|) = \left(\frac{R_s}{R_c}\right)^2 E(n_c). \quad (19)$$

Equations (18) and (19) yield:

$$E(|\text{cov}(t)|) \geq \left(\frac{R_s}{R_c}\right)^2 2m \geq \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor. \quad (20)$$

Therefore, the network is expected k-covering with $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$. ∎

Regarding lemma 2, increasing m in an expected m-connected WSN will also increase k, and, in turn, make the random WSN k-covering with $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$ in the asymptotic conditions. Therefore, the following result also holds. This result is assessed via experiments too.

Result 5. An m-connected WSN resulted by increasing m in a uniformly deployed expected m-connected WSN (i.e. in asymptotic conditions), will be k-covering with a probability tending to 1 such that $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$.

An EMK WSN can be straightly generated by uniformly deploying a WSN whose minimum number of nodes is calculated based on lemma 3.

Lemma 3. A uniformly deployed WSN with random targets is EMK, if the following relation holds for the number of nodes, i.e. $\boldsymbol{n}$:

$$n \geq \begin{cases} \left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1 & \forall k \leq \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor \\ \max\left(\left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1, \left\lceil \frac{kA}{A_{\min}^{\text{cov}}} \right\rceil \right) & \forall k > \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor \end{cases} \quad (21)$$

Proof. Firstly, let $k \leq \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$. Generally, a minimum number of $\left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1$ randomly scattered nodes yield an expected m-connected WSN based on result 1. According to lemma 2, this WSN is inherently expected k-connected with $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$. Therefore, for $k \leq \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$, the WSN is EMK.

Secondly, let $k > \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$.

According to results 3 and 1, a minimum number of $\left\lceil \frac{kA}{A_{min}^{cov}} \right\rceil$ and $\left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1$ random nodes yield expected k -covering and expected m-connected WSNs, respectively. Therefore, if $n \geq \max(\left\lceil \frac{A^2 m}{\alpha A' R_c^2} \right\rceil + 1, \left\lceil \frac{kA}{A_{min}^{cov}} \right\rceil)$, a uniformly deployed WSN will have both properties, hence, will be EMK. ∎

Algorithm 1. Structure of REMK.

1. ***Inputs***:
2. $AoI,\ T, m, k$;
3. $random\ EMK\ WSN$, $based\ on\ lemma$ 4.
4. ***Output***:
5. $V = final\ set\ of\ nodes\ in\ the\ output\ WSN$
6. **Body**:
7. V = set of nodes in the input EMK WSN
8. Foreach $t \in T$:
9. If $|cov(t)| < k$
10. $v' = \ Generate\ k - |cov(t)| random\ nodes\ in\ cov(t)$;
11. $V = V\ \cup v'$;
12. Endif
13. Endfor

Stage 1

14. $temp = V$;
15. Foreach $v \in temp$:
16. If $v \notin N(S)$ & $|\sup(v)| < m$
17. $v' = Generate\ m - |sup(v)|\ random\ nodes\ on\ \widehat{A_v^{\epsilon}}\ in\ the\ AoI$;
18. $V = V\ \cup v'$;
19. $temp = temp \cup v'$;
20. Endif
21. $remove\ v\ from\ temp$;
22. Endfor

Stage 2

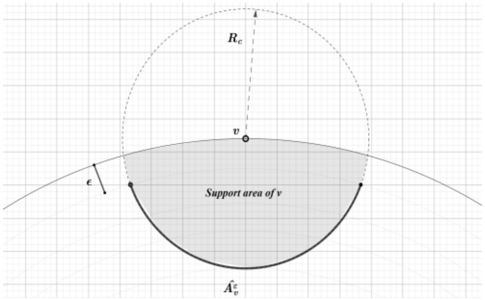

Figure 4. Arc $\widehat{A_v^{\epsilon}}$, the bolded solid curve, is the intersection of borders of communication and support areas of node v, with distance ϵ from the endpoints. It is the geometrical location where newly generated nodes at stage 2 of REMK are added to make node v m-connected.

9. Generating a Random MK WSN

In order to generate a random MK WSN, firstly, a random EMK WSN with the minimum number of nodes, based on lemma 3, is produced. Secondly, an algorithm, named REMK, is applied to turn it into an MK WSN. The structure of REMK is depicted in algorithm 1. It is composed of two stages. Stage 1 makes the input EMK WSN *k*-covering, while stage 2 makes it *m*-connected.

At stage 1, i.e., lines 7 to 13 of algorithm 1, a number of $k - |cov(t)|$ nodes are added to the neighborhood of each target t with $|cov(t)| < k$. Therefore we have $|cov(t)| \geq k$ for every target t at the end, and the WSN will be *k*-covering. All of the generated nodes are added to set V, which had already been initialized with the nodes of the input EMK WSN. At stage 2, the arc $\widehat{A_v^{\epsilon}}$ is defined as the intersection of borders of communication and support areas of every node v, with distance ϵ from the endpoints, as depicted in Figure 4. At this stage, i.e. lines 14 to 22 of algorithm 1, a number of $m - |\sup(v)|$ nodes are randomly generated at the arc $\widehat{A_v^{\epsilon}}$ for every node v, if $|\sup(v)| < m$. The reason is that this arch is the geometrical location of the furthest points in the support area of a node. Therefore, by adding the nodes on this arch, fewer number of nodes will be required to connect v to the sink. The newly generated nodes are added to set V, as well. Since $\widehat{A_v^{\epsilon}}$ is located at the support area of v, all of the newly generated nodes belong to $\sup(v)$. Therefore, we have $|\sup(v)| \geq m$ for every node at the end, and WSN will be *m*-connected according to lemma 1.

It should be noted that stage 2 will converge in a finite number of iterations since the distance of the newly generated nodes from the sink diminishes gradually. Two resulting MK WSNs are depicted in Figure 5.

10. Complexity Analysis of REMK

The time complexity of REMK is addressed as the total number of iterations of both loops since they are the most expensive steps, as follows:

The input EMK WSN has a minimum number of nodes n, based on lemma 3. Thus the complexity of the first stage is $\theta(|T|)$. An upper bound of $k|T|$ nodes will be generated at this stage if the cover sets of all targets are empty. Accordingly, there will be an upper bound of $n + k|T|$ nodes at the end/start of stage1/stage 2. The worst-case for stage 2 occurs when all nodes are located at the brim of the deployment area, without being neighbors to each other and with empty cover sets.

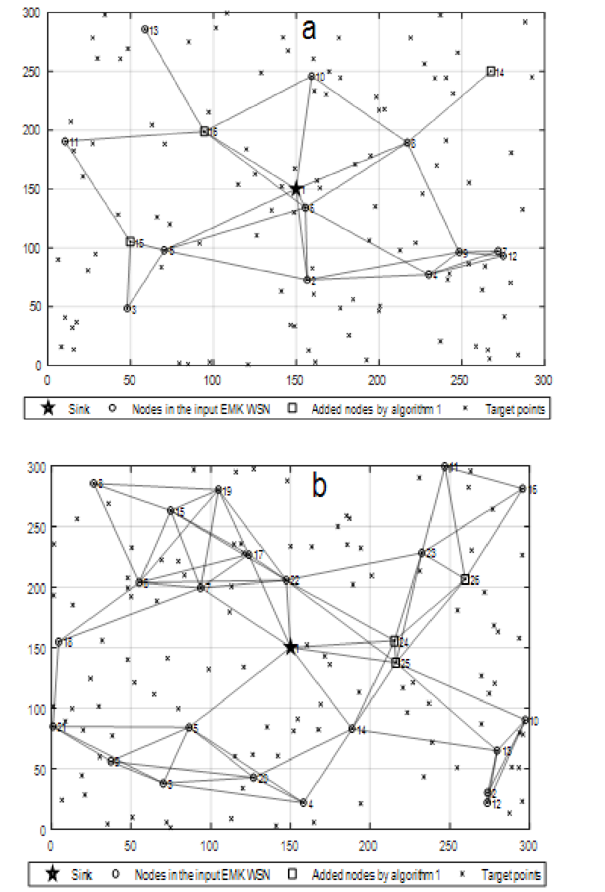

Figure 5. Output of REMK for a square AoI of length 300, $R_c = R_s = 100$, and a) m = 1, k = 1, and b) m = 2, k = 2. Firstly, an EMK WSN is generated, and secondly, it is reformed to an MK WSN by REMK.

In this case, for a node, namely v in Figure 6, m nodes are generated at $\widehat{A_v^\epsilon}$ with the furthest possible distance from the sink, i.e. point l_1 in the figure. Similarly, a number of m neighbors are generated for the nodes at l_1, i.e. point l_2 in Figure 6. This process is repeated in the following rounds until all newly generated nodes are placed at the neighborhood of the sink, leading to the sequence of points $l_1, l_2, \dots l_h$. Therefore, the upper bound of the number of newly generated nodes will be hm for a single initial node. Since there are at most $n + k|T|$ initial nodes at the start of stage 2, an upper bound for the number of nodes in set V will be $(n + k|T|)hm + (n + k|T|)$ at the end of stage 2. Since the second loop iterates once for each node in V, the worst-case complexity of the second stage will be $O((n + k|T|)(hm + 1))$. Parameter h is constant for a given AoI, R_c, and ϵ. Having summed up and simplified the complexity terms of both stages, the worst-case complexity of REMK will be $O(nm + k|T|m)$, i.e. a polynomial complexity.

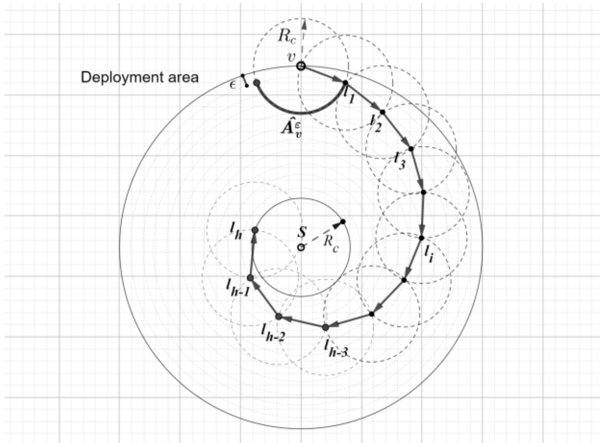

Figure 6. Worst case of REMK for a single node, namely v. In the worst case, newly generated nodes will be added at points $l_i, i = 1, \dots, h$.

10.1. Average-case complexity

The average-case complexity is studied in asymptotic conditions. The asymptotic conditions take place when k or m grow larger in the input EMK WSN. In such situations, the polynomial worst-case complexity of REMK would not happen, thanks to the initial input EMK WSN. That is, according to results 2 and 4, the input EMK WSN tends to be both m-connected and k-covering with a probability tending to 1, asymptotically. This fact, in turn, reduces the average-case complexity of REMK to $\theta(|T| + n)$ since no more nodes are added to the input EMK WSN at stages 1 or 2 of REMK. Likewise, the number of nodes in the output MK WSN will remain intact and of linear order asymptotically (i.e. $|V| \in \theta(n)$). The experimental studies confirm this analysis as well.

11. Experimental Studies and Simulation Results

The simulations are logically divided into two parts. Firstly, in sections 11.1 to 11.4, the network characteristics are studied by assessing the theoretical results through simulations. In these experiments, a circular AoI of diameter 300 is considered centered at the sink. This way, the border effect exerted on the support areas is voided since all of the support areas fall within the AoI. Also the minimum coverage area, i.e. A_{min}^{cov}, can be calculated similar to (5) since it is the intersection of two circular areas (AoIs of toroidal shapes are common to void the border effects [26]). Secondly, in section 11.5, the performances of some metaheuristic approaches for solving the minimum MK WSN problem have been benchmarked by REMK. In this experiment, a square AoI of length 300 is deployed. The AoI is centered at the sink. The targets and nodes are deployed uniformly across AoI in both parts. Table 3 summarizes the incorporated parameters. Simulations are carried out via MATLAB. The specifications of the underlying system have been Intel(R) Core(TM) i7 CPU @ 4GHz with 16 GB RAM and windows 10.

Table 3. Incorporated parameters.

Parameter	Value
$\boldsymbol{Network\ diameter}$	300
$\boldsymbol{\epsilon}$	0.01
$\boldsymbol{\lvert T \rvert}$	100
$\boldsymbol{R_c}$	100
$\boldsymbol{R_s}$	75, 100, 125
$\boldsymbol{m, k}$	1,…, 200

11.1. Empirical network characteristics

In this experiment, the m-connectivity and k-coverage characteristics of random WSNs, and EKM WSNs are studied.

Firstly, the empirical support and cover set sizes in random WSNs of various sizes are studied. A set of random networks was generated for each network size, and the observed average size of support and cover sets was calculated. Empirical average support and cover set sizes have been compared with their minimum expected theoretical values based on equations (11) and (17), respectively, in figure 7. As it can be seen, the empirical average is always greater than or equal to the expected theoretical minimum; hence, Equations (11) and (17) hold.

To assess results 2 and 4, several expected m-connected and expected k-covering WSNs were generated with various values of R_c and R_s. For this purpose, the minimum number of required nodes for an expected m-connected and k-covering WSN was calculated based on results 1 and 3, respectively. Then the nodes were spread uniformly across AoI. The percentage of m-connected nodes and k-covered targets are depicted in Figure 8. As it can be seen, the ratio of k-covered and m-connected nodes increases by increasing k and m, respectively. Hence, the outcomes are in compliance with results 2 and 4. Efficiency of EMK WSNs is based on lemma 3. The calculated size of each EMK WSN is partially depicted in Figure 9. The efficiency of the generated EMK WSNs is measured in terms of the average number of violating nodes and targets (i.e. nodes/targets that are not m-connected/k-covered). The results are depicted in Figure 10 and Figure 11. As it can be seen, by increasing k and m, the number of violating nodes and targets decrease (it should be noted that the number of violating targets has a dwindling trend, though with a low rate). This result also complies with results 2, 4, and the discussion of section 10.1.

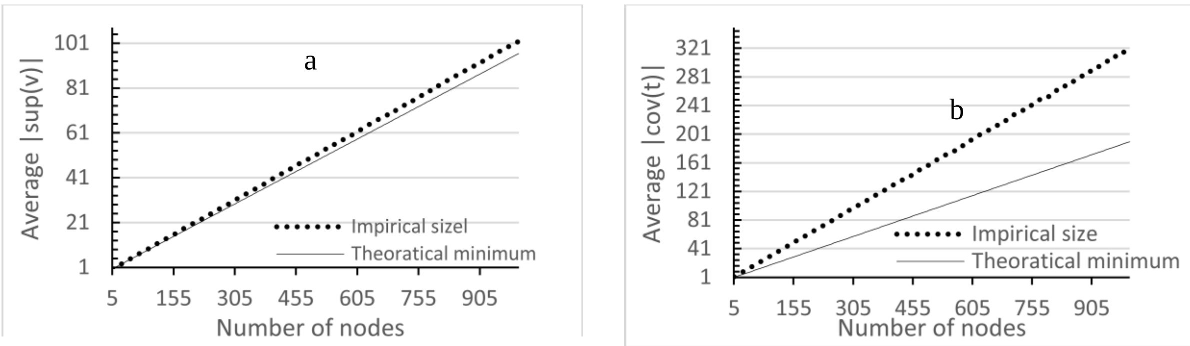

Figure 7. Comparing theoretical minimum and empirical expectations of a) support and b) cover set sizes.

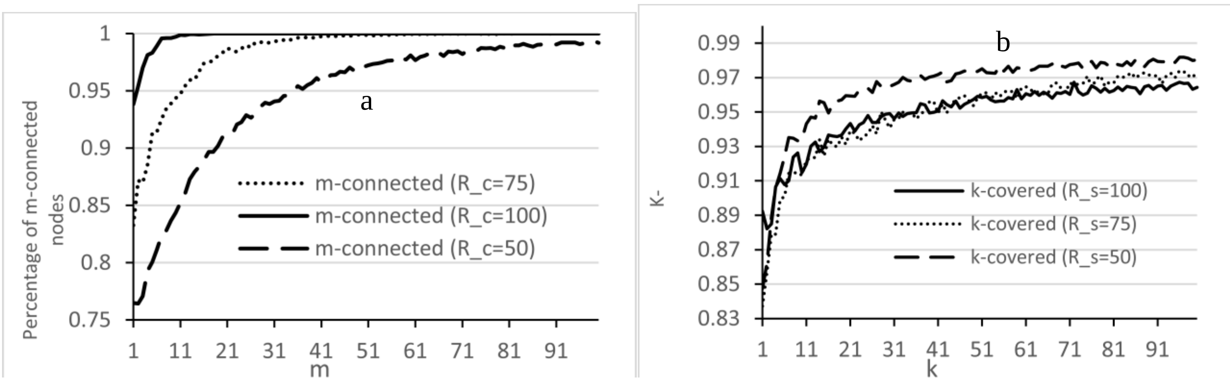

Figure 8. Percentage of m-connected nodes, and k-covered targets for various sensing and communication ranges, in a) expected m-connected WSN, and b) expected k-covering WSN, respectively. Number of nodes is calculated based on results 1 and 3.

m \ k	1	2	3	4	5	6	7	8	9	10
1	12	12	16	22	27	32	37	43	48	53
2	22	22	22	22	27	32	37	43	48	53
3	33	33	33	33	33	33	37	43	48	53
4	43	43	43	43	43	43	43	43	48	53
5	53	53	53	53	53	53	53	53	53	53
6	64	64	64	64	64	64	64	64	64	64
7	74	74	74	74	74	74	74	74	74	74
8	84	84	84	84	84	84	84	84	84	84
9	95	95	95	95	95	95	95	95	95	95
10	105	105	105	105	105	105	105	105	105	105

Figure 9. Minimum number of required nodes for an EMK WSN, calculated based on lemma 3 (equation (21)).

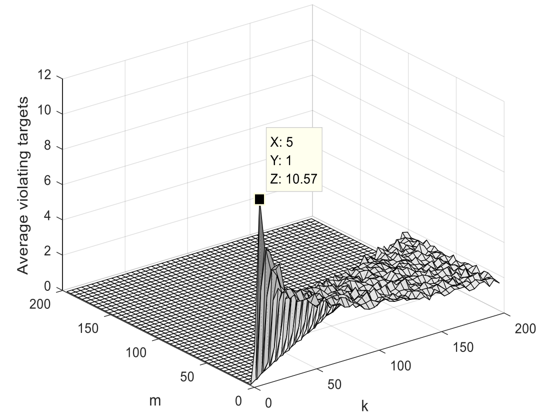

Figure 10. average number of violating targets in EMK WSN.

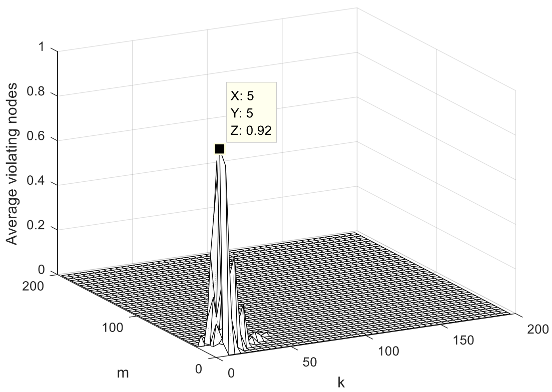

Figure 11. Average number of violating nodes in EMK WSN.

11.2. Size of Generated MK WSN

The average number of nodes in an *m*-connected *k*-covering WSN, generated by REMK is depicted in Figure 12 for various values of m and k. For a precise analysis, a set of EMK WSNs was generated for each specific pair (k, m), and the results were averaged. A comparison with the number of nodes in the initial EMK WSN (i.e. Figure 9) unveils that a limited number of nodes are added by REMK. In order to further study this issue, the total amount of augmented nodes by REMK is depicted in Figure 13.a, for a broad range of k and m. As it can be seen, by increasing m for a fixed k, the EMK WSN becomes *m*-connected *k*-covering since no further nodes have been added. Also by increasing k for a fixed m, a limited number of nodes (less than 13) have been added. Figure 13.b pictures the average number of added nodes for $m = 1,5,10,20$, and various values of k, more precisely. The number of added nodes fluctuates around an average of 9.40, with variance 0.88, for $k > 80$. This result shows that for a large enough k and m, most of the initially generated nodes and targets are *m*-connected and *k*-covering, respectively. Thus a scalar number of nodes has been added by REMK, just to reform a limited number of violating nodes/targets.

11.3. Relationship between *m*-connectivity and *k*-coverage

To assess lemma 2, several expected *m*-connected WSNs were randomly generated, and their *k*-coverage property was studied. For this purpose, the average size of the cover sets was calculated empirically for each expected *m*-connected network. Also the theoretical anticipated value of $|cov(t)|$ was calculated based on lemma 2. The results obtained are compared in Figure 14. As it can be seen, the empirical mean of $|cov(t)|$ is always greater than or equal to its theoretical approximation for various values of R_s; this result is in keeping with lemma 2, i.e., the expected *m*-connected WSNs are also expected *k*-connected with $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$, for large enough m. A further assessment was conducted based on result 5, in which the relation between *m*-connectivity and *k*-coverage was studied empirically. For this purpose, a number of expected *m*-connected WSNs were randomly generated by increasing m from 1 to 50. Then the approximate number of required nodes to turn the EMK WSN into an MK WSN was calculated by REMK, as an inclusive measure of *m*-coverage and *k*-connectivity properties.

m \ k	1	2	3	4	5	6	7	8	9	10
1	13.15	15.02	19.52	26.03	31.22	36.56	42.19	48.01	53.26	58.64
2	23.07	23.34	24.19	26.08	31.02	36.48	42.08	48.11	53.08	58.51
3	34.00	34.01	34.06	34.52	35.74	36.92	42.20	47.76	52.94	58.93
4	44.06	44.07	43.93	44.16	44.24	45.10	46.48	48.36	52.99	57.94
5	53.92	53.77	53.95	53.91	53.81	54.06	54.14	55.52	56.99	58.60
6	64.60	64.67	64.59	64.63	64.75	64.87	64.87	65.01	65.66	65.98
7	74.67	74.61	74.54	74.55	74.65	74.70	74.63	74.66	74.58	75.04
8	84.39	84.59	84.49	84.57	84.67	84.50	84.47	84.39	84.48	84.71
9	95.40	95.32	95.43	95.58	95.25	95.44	95.32	95.29	95.30	95.68
10	105.28	105.32	105.45	105.19	105.24	105.32	105.38	105.22	105.09	105.66

Figure 12. Average number of nodes in an *m*-connected *k*-covering WSN, generated by REMK.

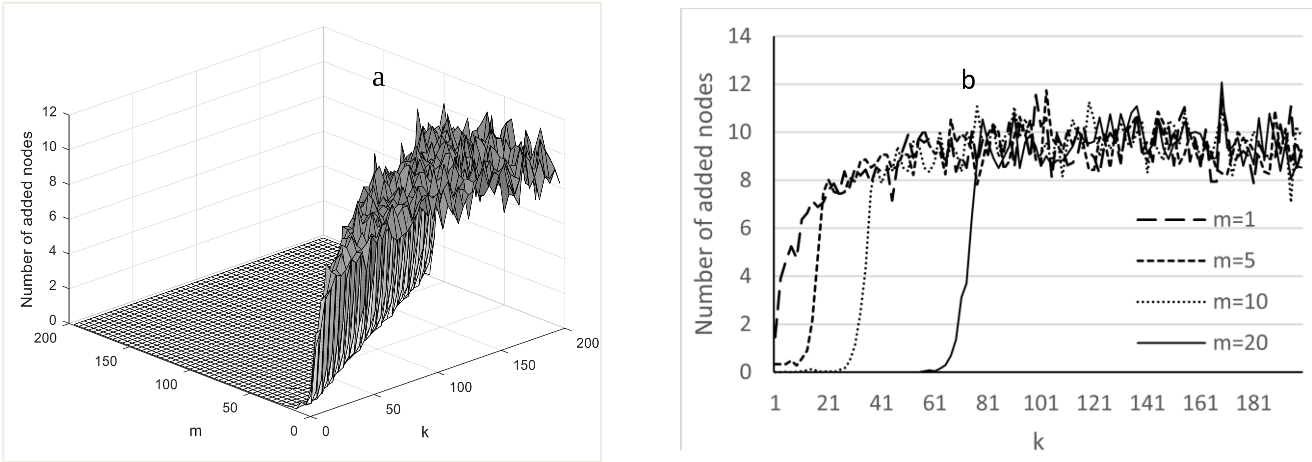

Figure 13. average number of nodes added to input random EMK WSN by REMK for a) a broad range of m, and k, and b) m = 1, 5, 10, 20 and various values of k.

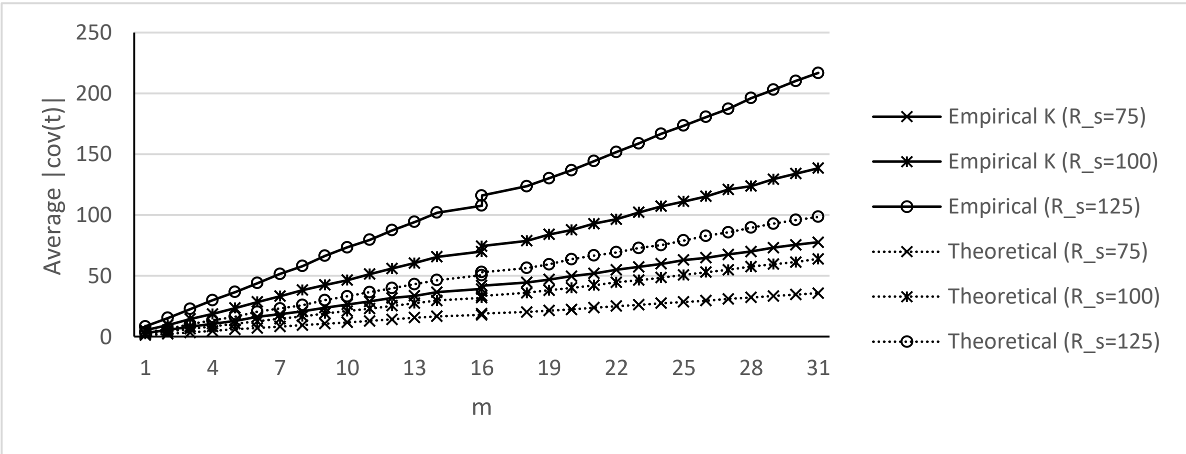

Figure 14. Relation between m and average size of cover sets, for $R_c = 100$ and $R_s = 75, 100, and\ 125$.

The color maps of Figure 15 depict the results for $R_c = 100$ and $R_s = 75, 100$, and 125. Each point in the diagrams depicts the number of required nodes. The darker the point is, the more nodes are required, e.g. white points show an MK WSN. A solid line sketches the frontier between MK WSNs and non-MK WSNs. The frontier lines are in outright compliance with result 5. Based on this result, the slope of the discriminating border between MK WSNs and non-MK WSN should have been $\frac{1}{2}\left(\frac{R_c}{R_s}\right)^2$, resulting in anticipated slopes of 0.89, 0.50, and 0.32 for sensing ranges 75, 100, and 125, respectively. Based on the empirical results, the slopes of the frontier line have been 0.84, 0.54, and 0.39, respectively, which are very close to their theoretical counterparts.

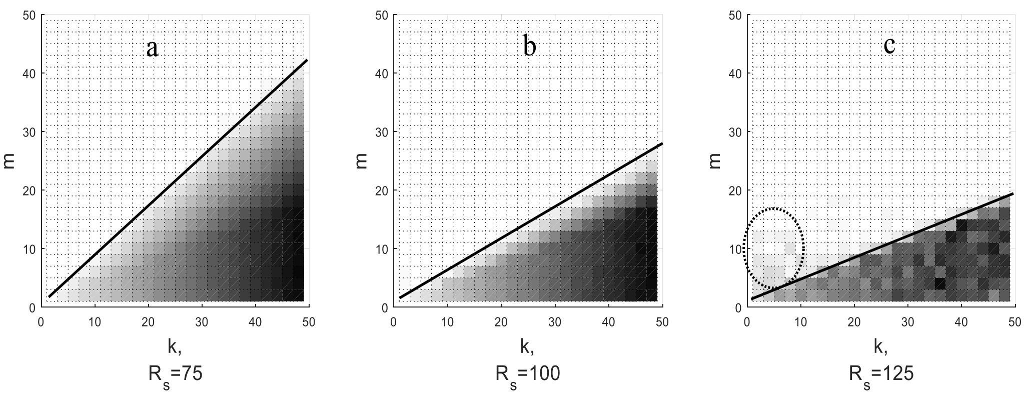

Figure 15. Relation between m-connectivity and k-coverage for $R_c = 100$, and a) $R_s = 75$, b) $R_s = 100$, and c) $R_s = 125$. Darkness of points shows amount of added nodes to EMK WSN by REMK.

As an instance, consider $R_c = R_s = 100$. Based on result 5, when $R_s = R_c$, it is anticipated with a high probability that an expected m-connected WSN becomes m-connected and k-covering with $k = 2m$, for a sufficiently large m; in keeping with this analysis, as Figure 15.c conveys, no more reformation by adding nodes is required when $m \gtrapprox \frac{1}{2}k$ in practice too. The same interpretation exists for the other two cases. It should be noted that there are some jitters for small values of k (e.g. the dotted ellipse in Figure 15.c), which is diminished by the growth of either k or m.

11.4 Empirical complexities

As it can be seen in Figure 13, a scalar number of nodes have been added to the input EMK by REMK (a maximum of 12.07 on average) for various values of m and k. Therefore, the time complexity of REMK has been linear empirically. Also evident from Figure 10 and Figure 11, by increasing m and k, there will be no further violating nodes or targets. This result proves that in asymptotic conditions, the EMK WSN tends to be both *m*-connected and *k*-covering from scratch; hence, no further node is added to WSN by REMK, and the average-case complexity will be $\theta(n + |T|)$. Likewise, the size of the network will be $\theta(n)$, where n is calculated through lemma 3. This issue is in compliance with the discussion of section 10.1.

The empirical time overhead of REMK is depicted in Figure 16 for $R_c = R_s = 100$, and various values of m and k. Fifty random EMK WSNs were generated for each pair of *m* and *k*, and the time overheads were averaged. As shown, the time overhead has been less than 15 ms for *m* = 100 and *k* = 100. Moreover, the approximately constant gradient of the diagram for large enough m and k approves the linear average time complexity, as discussed in section 10.1. The reason is that EMK WSNs become MK with a probability tending to 1 asymptotically.

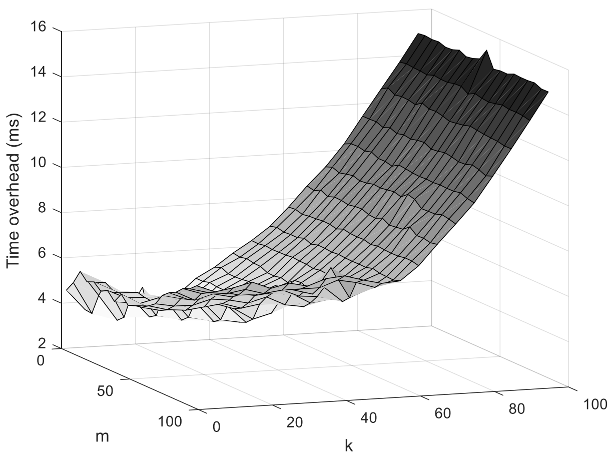

Figure 16. Computational time overhead of REMK for $R_c = R_s = 100$ and $m, k \in \{1, \ldots, 100\}$.

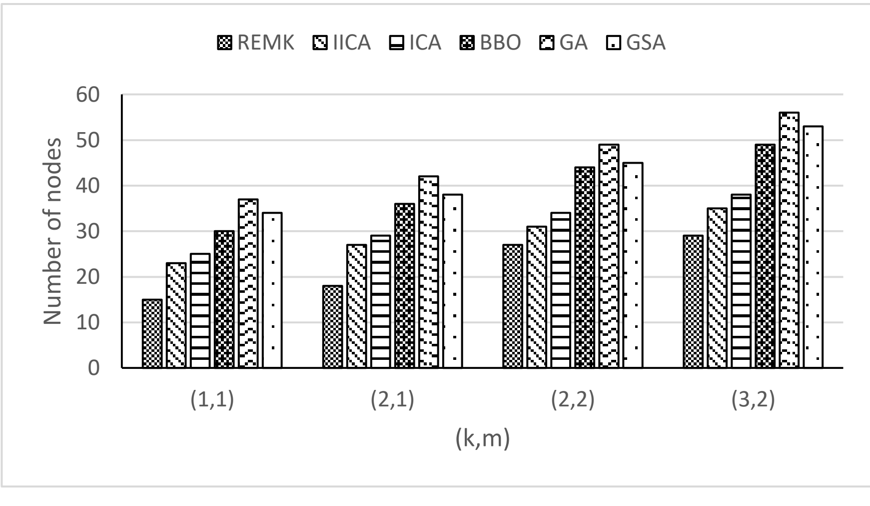

Figure 17. Comparison of various methods regarding number of nodes in MK WSNs.

11.5. Benchmarking some meta-heuristic approaches

Even though the presented work aims not to solve the np-complete problem of *minimum MK WSNs*, the performance of some metaheuristic approaches for this problem has been benchmarked to reveal the suitable size of the generated random MK WSNs. For this purpose, REMK is compared with some approaches based on ICA [15], IICA [16], BBO [6], GA [18], and GSA [19] in terms of the number of nodes. In all of the benchmarked approaches, a number of initial potential positions for sensor node placement are generated randomly. This potential WSN must be taken m-connected and k-covering for granted. Therefore, many potential positions are generated to make sure that the potential WSN is MK. Then a subset of the potential positions is selected by minimizing some objective functions such that m-connectivity and k-coverage properties are preserved.

The results for the typical values of $m = 1,2$, and $k = 1, 2, 3$ are depicted in Figure 17. For each pair of (m,k) the number of nodes resulting from 50 trials has been averaged and rounded up. The variances have been less than 0.01. As it can be seen, in such cases, the number of nodes resulting from the metaheuristic approaches is greater than those of the proposed random system. The reason is that the metaheuristic methods are susceptible to local extremums. The above-mentioned benchmarked approaches typically choose many potential positions uninformedly since a fair number is unknown a priory. Hence, they return suboptimal local extremums. It is exacerbated when there are many unknown variables, i.e. potential positions. As the results show, the proposed method can yield a rule for generating random MK WSNs with a fair number of nodes. The metaheuristic methods can further process this initial random MK WSN.

12. Conclusion

This paper proposed EMK WSNs as a building block for generating random MK WSNs. The concept of m-connectivity was founded based on the *support sets* to guarantee the existence of m disjoint paths between each pair of nodes. We showed that EMK WSNs tended to MK WSNs in asymptotic conditions and delivered a probabilistic algorithm of average-case and worst-case complexities of orders $\theta(n + |T|)$ and $O(nm + k|T|m)$, to turn an EMK WSN into MK, where n is the size of the input EMK WSN, and $|T|$ is the number of targets. A lower bound on the size of an EMK WSN was calculated as a function of k, m, and some network characteristics (i.e. AoI and communication and sensing ranges). It turned out that an expected m-connected WSN is also expected k-covering with $k = \left\lfloor 2m\left(\frac{R_s}{R_c}\right)^2 \right\rfloor$, and this relation holds in asymptotic conditions in which the network becomes m-connected, and hence k-covering. The empirical studies confirmed the theoretical results as well.

References

[1] S. Messaoud, A. Bradai, S. H. R. Bukhari, P. T. A. Qung, O. B. Ahmed, and M. Atri, “A survey on machine learning in internet of things: Algorithms, strategies, and applications,” *Internet of Things*, p. 100314, 2020.

[2] C. Goumopoulos and I. Mavrommati, “A framework for pervasive computing applications based on smart objects and end user development,” *Journal of Systems and Software*, Vol. 162, p. 110496, 2020.

[3] H. Sharma, A. Haque, and F. Blaabjerg, “Machine Learning in Wireless Sensor Networks for Smart Cities: A Survey,” *Electronics*, Vol. 10, No. 9, p. 1012, 2021.

[4] H. Fouad, N. M. Mahmoud, M. S. El Issawi, and H. Al-Feel, “Distributed and scalable computing framework for improving request processing of wearable IoT assisted medical sensors on pervasive computing system,” *Computer Communications*, Vol. 151, pp. 257-265, 2020.

[5] A. Boulmaiz, N. Doghmane, S. Harize, N. Kouadria, and D. Messadeg, “The use of WSN (wireless sensor network) in the surveillance of endangered bird species,” *in Advances in Ubiquitous Computing: Elsevier*, 2020, pp. 261-306.

[6] G. P. Gupta and S. Jha, “Biogeography-based optimization scheme for solving the coverage and connected node placement problem for wireless sensor networks,” *Wireless Networks*, Vol. 25, No. 6, pp. 3167-3177, 2019.

[7] H. P. Gupta, P. K. Tyagi, and M. P. Singh, “Regular node deployment for $ k $-coverage in $ m $-connected wireless networks,” *IEEE Sensors Journal*, Vol. 15, Nno. 12, pp. 7126-7134, 2015.

[8] H. P. Gupta, S. V. Rao, and T. Venkatesh, “Analysis of stochastic coverage and connectivity in three-dimensional heterogeneous directional wireless sensor networks,” *Pervasive and Mobile Computing*, Vol. 29, pp. 38-56, 2016.

[9] Y. Wang, S. Wu, Z. Chen, X. Gao, and G. Chen, “Coverage problem with uncertain properties in

wireless sensor networks: A survey," *Computer Networks*, Vol. 123, pp. 200-232, 2017.

[10] M. Mansour and F. Jarray, "An iterative solution for the coverage and connectivity problem in wireless sensor network," *Procedia Computer Science*, Vol. 63, pp. 494-498, 2015.

[11] W.-C. Ke, B.-H. Liu, and M.-J. Tsai, "Constructing a wireless sensor network to fully cover critical grids by deploying minimum sensors on grid points is NP-complete," *IEEE Transactions on Computers*, Vol. 56, No. 5, pp. 710-715, 2007.

[12] S. Harizan and P. Kuila, "Nature-inspired algorithms for k-coverage and m-connectivity problems in wireless sensor networks," *in Design Frameworks for Wireless Networks: Springer*, 2020, pp. 281-301.

[13] S. M. Hosseinirad, "Multi-layer clustering topology design in densely deployed wireless sensor network using evolutionary algorithms," *Journal of AI and Data Mining*, Vol. 6, NO. 2, pp.297-311, 2018.

[14] H. M. Ammari, "Connected k-coverage in two-dimensional wireless sensor networks using hexagonal slicing and area stretching," *Journal of Parallel and Distributed Computing*, Vol. 153, pp. 89-109, 2021.

[15] H. Sheikhi and W. Barkhoda, "Solving the k-coverage and m-connected problem in wireless sensor networks through the imperialist competitive algorithm," *Journal of Interconnection Networks*, Vol. 20, No. 01, p. 2050002, 2020.

[16] W. Barkhoda and H. Sheikhi, "Immigrant imperialist competitive algorithm to solve the multi-constraint node placement problem in target-based wireless sensor networks," *Ad Hoc Networks*, Vol. 106, p. 102183, 2020.

[17] J. Chelliah and N. Kader, "Optimization for connectivity and coverage issue in target - based wireless sensor networks using an effective multiobjective hybrid tunicate and salp swarm optimizer," *International Journal of Communication Systems*, Vol. 34, No. 3, p. e4679, 2021.

[18] S. K. Gupta, P. Kuila, and P. K. Jana, "Genetic algorithm approach for k-coverage and m-connected node placement in target based wireless sensor networks," *Computers & Electrical Engineering*, Vol. 56, pp. 544-556, 2016.

[19] C. Jehan and D. S. Punithavathani, "Potential position node placement approach via oppositional gravitational search for fulfill coverage and connectivity in target based wireless sensor networks," *Wireless Networks*, Vol. 23, No. 6, pp. 1875-1888, 2017.

[20] P. Natarajan and L. Parthiban, "k-coverage m-connected node placement using shuffled frog leaping: Nelder–Mead algorithm in WSN," *Journal of Ambient Intelligence and Humanized Computing*, pp. 1-16, 2020.

[21] V. K. Akram, O. Dagdeviren, and B. Tavli, "A Coverage-Aware Distributed k-Connectivity Maintenance Algorithm for Arbitrarily Large k in Mobile Sensor Networks," *IEEE/ACM Transactions on Networking*, 2021.

[22] V. Khalilpour Akram, Z. Akusta Dagdeviren, O. Dagdeviren, and M. Challenger, "PINC: Pickup Non-Critical Node Based k-Connectivity Restoration in Wireless Sensor Networks," *Sensors*, Vol. 21, No. 19, p. 6418, 2021.

[23] R. Ferrero, M. V. Bueno-Delgado, and F. Gandino, "In-and out-degree distributions of nodes and coverage in random sector graphs," *IEEE transactions on wireless communications*, Vol. 13, No. 4, pp. 2074-2085, 2014.

[24] M. Khanjary, M. Sabaei, and M. R. Meybodi, "Critical density for coverage and connectivity in two-dimensional aligned-orientation directional sensor networks using continuum percolation," *IEEE Sensors Journal*, Vol. 14, No. 8, pp. 2856-2863, 2014.

[25] M. Khanjary, M. Sabaei, and M. R. Meybodi, "Critical density for coverage and connectivity in two-dimensional fixed-orientation directional sensor networks using continuum percolation," *Journal of Network and Computer Applications*, Vol. 57, pp. 169-181, 2015.

[26] H. P. Gupta, S. V. Rao, and T. Venkatesh, "Critical sensor density for partial coverage under border effects in wireless sensor networks," *IEEE transactions on wireless communications*, Vol. 13, No. 5, pp. 2374-2382, 2014.

[27] Z. Yu, J. Teng, X. Li, and D. Xuan, "On wireless network coverage in bounded areas," *in 2013 Proceedings IEEE INFOCOM, 2013: IEEE*, pp. 1195-1203.

[28] R. Tan, G. Xing, B. Liu, J. Wang, and X. Jia, "Exploiting data fusion to improve the coverage of wireless sensor networks," *IEEE/ACM Transactions on networking*, Vol. 20, No. 2, pp. 450-462, 2011.

[29] T. Böhme, F. Göring, and J. Harant, "Menger's theorem," *Journal of Graph Theory*, Vol. 37, No. 1, pp. 35-36, 2001.

یک رویکرد تصادفی برای پیاده سازی شبکه های حسگر بیسیم *m*-همبند و *k*-پوشش

وحید قاسمی[۱] و علی قنبری سرخی[۲،*]

[۱] گروه مهندسی کامپیوتر، دانشکده فناوری اطلاعات، دانشگاه صنعتی کرمانشاه، کرمانشاه، ایران.

[۲] گروه مهندسی کامپیوتر، دانشکده مهندسی برق و کامپیوتر، دانشگاه علم و فناوری مازندران، بهشهر، ایران.

ارسال ۲۰۲۲/۰۲/۲۲؛ بازنگری ۲۰۲۲/۰۴/۲۴؛ پذیرش ۲۰۲۲/۰۵/۰۸

چکیده:

پیاده‌سازی شبکه های حسگر بیسیم m-همبند و k-پوشش (MK) برای ارسال بسته‌ها و پوشش اهداف از اهمیت ویژه‌ای برخوردار است. در این مقاله، پیاده سازی شبکه‌های حسگر MK تصادفی بر مبنای *شبکه‌های* m-*همبند و* k-*پوشش مورد انتظار* (EMK) پیشنهاد شده است. شبکه‌های حسگر بیسیم EMK شبکه‌هایی تصادفی هستند که در آنها امید ریاضیِ درجه همبندی و پوشش به ترتیب برابر با m و k است. پیاده‌سازی شبکه‌های حسگر بیسیم EMK با اثبات رابطه‌ای بین m-همبندی و k-پوشش بودن، و نیز محاسبه یک کران پایین برای تعداد گره‌های شبکه انجام شده است. همچنین، نشان داده شده که شبکه‌های EMK در حالت مجانبی به شبکه‌های MK میل می‌کنند. برای تبدیل شبکه‌های EMK به MK در شرایط غیر مجانبی، الگوریتمی با پیچیدگی چند جمله‌ای در بدترین حالت، و خطی در حالت متوسط ارائه گردیده است. ویژگی *m*-همبند بودن بر مبنای مجموعه‌های پشتیبان تعریف شده تا وجود m مسیر مجزا بین هر گره و چاهک تضمین شود. نتایج نظری با پیاده‌سازی‌های متعدد ارزیابی شده، و چندین رویکرد فرا-اکتشافی برای نشان دادن اندازه مناسب شبکه‌های MK تولید شده، مورد مقایسه قرار گرفته‌اند.

کلمات کلیدی: m-همبندی، k-پوشش، شبکه‌های حسگر بیسیم، مجموعه پشتیبان.