

## Modeling Length of Hydraulic Jump on Sloping Rough Bed using Gene Expression Programming

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### Abstract

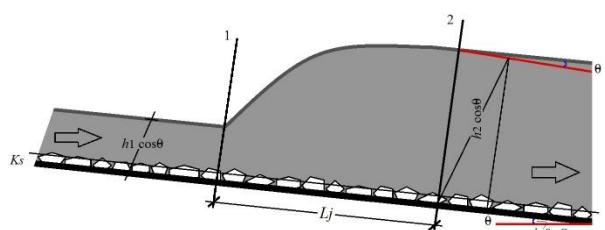
Determination of the hydraulic jump characteristics can play a crucial role to design an optimized stilling basin or other facilities at downstream of the hydraulic jump. In other words, the length of hydraulic jump is one the most important parameters involved to obtain the length of the structures. In contrast, the artificial intelligence techniques have been extensively applied to simulate various hydraulic problems such as hydraulic jump features. In this work, the length of hydraulic jump on sloping rough beds was predicted using the Gene Expression Programming (GEP) model for the first time because the GEP model could provide an explicit equation to estimate the target function. The Monte Carlo simulations were used to enhance the capability of the GEP model. In addition, k-fold cross-validation was employed in order to verify the results of the GEP model. In order to determine the length of hydraulic jump, five different GEP models were introduced using the input parameters. Then by analyzing the GEP model results, the superior model was presented. For the superior model, the correlation coefficient, mean absolute percentage error, and root mean square error were computed to be 0.901, 11.517, and 1.664, respectively. According to the sensitivity analysis, the Froude number at upstream of hydraulic jump was identified as the most important parameter to model the length of hydraulic jump. Furthermore, the Partial Derivative Sensitivity Analysis (PDSA) was performed. For instance, PDSA was calculated as positive for all the input variables.

**Keywords:** *Length of Hydraulic Jump, Sloping Rough Bed, Sensitivity Analysis, Gene Expression Program, Partial Derivative Sensitivity Analysis.*

### 1. Introduction

Hydraulic jumps are usually accompanied by the turbulence and rapid transformation of flow regime from subcritical to supercritical. This phenomenon occurs at the downstream of the structures such as slide gates, perpendicular weirs or ogee spillways. Generally, beds with hydraulic jump occurring on them are sloping and rough. On the other hand, determination of the hydraulic jump length ( $L_j$ ) is necessary for a precise design of the structures such as detention ponds. In figure 1, the schematic representation of hydraulic jump on the bed is illustrated.

Due to the importance of hydraulic jump, many experimental, analytical, and numerical studies have been carried out on this phenomenon by various researchers.



**Figure 1. Schematic representation of hydraulic jump formation on rough sloping beds**

For example, Rajaratnam [1] was one of the first ones who investigated the hydraulic jump behavior on rough beds. He showed that the presence of the rough bed had a significant impact on the hydraulic jump length reduction. Hughes and Flack [2] have conducted an experimental study regarding the hydraulic jump occurrence on rough beds. The analysis of their results showed

that the reduction of the hydraulic jump length depended on the upstream Froude number and the roughness height of the bed. Also Mohammed Ali [3], by conducting an experimental study, has investigated the effects of roughness in the rectangular form on the hydraulic jump characteristics. The range of the Froude number in his study varied from 4 to 10. He concluded that the applied roughness decreased the hydraulic jump length down to 27-67%. After that, Ead and Rajaratnam [4] have conducted an experimental study on the behavior of the hydraulic jump on rough beds with labyrinth roughness. In their study, the range of the upstream Froude number varied from 4 to 10. Ead and Rajaratnam [4] have investigated the hydraulic jump characteristics for three relative roughness conditions including 0.25, 0.43, and 0.5, and proved that the length of the hydraulic jump on the rough bed was half the hydraulic jump length on the smooth bed. Carollo *et al.* [5] have studied the hydraulic jump in rectangular canals for rough and smooth beds though an experimental research work. By analyzing the results of their study, they proposed a relationship for calculating the roller length on rough beds. Elsebaie and Shabayek [6] have also carried out an experimental study by the range of Froude number between 3 and 7.5 on five types of rough beds including sine, triangular, trapezoid with two different rectangular slopes. Their results indicated the hydraulic jump reduction on rough beds compared to smooth beds. Ahmed *et al.* [7] have experimentally investigated the effects of rough beds on the hydraulic jump characteristics. They suggested a relationship in terms of the flow Froude number for calculating the roller length. Nissi and Shafaei Bajestan [8], by conducting an experimental study for Froude numbers between 4.9 and 12.4, have examined the effects of rhombus roughness on the behavior of hydraulic jumps occurring in detention ponds. By analyzing the mentioned model results, they showed that the rough bed decreased hydraulic jump length down to 41%.

In the recent decades, the various artificial intelligence methods have been broadly used for modeling complex engineering phenomena. Also several studies have been carried out on the application of artificial intelligence techniques for modeling different problems [9-14]. Omid *et al.* [15] have modeled the hydraulic jump characteristics such as the jump length and the sequent depth ratio in rectangular channels using the artificial neural network. Naseri and Othman [16], using the artificial neural network (ANN), have evaluated the hydraulic jump characteristics

in rectangular channels. They modeled the hydraulic jump length for the Froude number range between 1.7 and 19.5. Karbasi and Azamathulla [17] have modeled the sequent depth ratio and the roller length of the hydraulic jump on rough beds using the gene expression programming model. They compared the mentioned model results with ANN and support vector regression and showed that the gene expression programming model simulated the hydraulic jump characteristics with a higher accuracy.

Regarding the literature, the soft computing techniques and artificial intelligence methods have been used to estimate the hydraulic jump because these tools are inexpensive, accurate, and fast. Besides, the length of hydraulic jump is utilized to determine the optimized length of the stilling basin at downstream of the hydraulic jump. Indeed, natural channels and hydraulic structures have sloping rough beds. However, a vast majority of the conducted studies on hydraulic jump have a smooth bed with an insignificant slope. Moreover, the length of hydraulic jump is an important parameter involved to estimate the length of the stilling basins or other hydraulic structures. Thus in the current work, the length of hydraulic jump in a sloping rough channel was simulated using the gene expression programming models, and then an explicit equation was presented to estimate this parameter.

Therefore, further investigations should be conducted on modeling the length of hydraulic jumps occurring on rough sloping beds. In this work, using the gene expression programming (GEP), the hydraulic jump length in a rectangular channel with a rough sloping bed was modeled for the first time.

Furthermore, this model is a robust tool that can simulate different problems. Additionally, the model provides an explicit equation to estimate the target function (length of hydraulic jump). For introducing the superior GEP model and identification of the parameter effective on the hydraulic jump length, five GEP models are defined. Then the superior model is identified by analyzing the results of the mentioned models, and a relationship is provided for computing the length of hydraulic jumps occurring on rough beds. Finally, the influence of each input parameter on the objective function (hydraulic jump length) is examined using the sensitivity analysis.

## 2. Methods and Materials

### 2.1. Gene Expression Programming (GEP)

There are numerous artificial intelligence methods and soft computing tools to simulate the hydraulic problems like the hydraulic jump on the sloping rough beds. It is worth noticing that GEP owns many advantages. For instance, it is quite accurate and fast, and can simulate linear and non-linear issues. In addition, this model can provide an explicit equation to calculate the length of hydraulic jump in the sloping rough flumes.

GEP is an evolutionary artificial intelligence method provided for the first time by Ferreira [18]. The basic difference between GEP, the genetic algorithm (GA), and the genetic programming is in the nature of the individuals. In GA, the individuals are as strings with fixed lengths (chromosomes), and in genetic programming, the individuals have different shapes and sizes (decomposition tree). However, in GEP, the individuals are as linear strings with fixed lengths (chromosomes) that express a non-linear nature and various sizes. In fact, GEP uses the advantages of both GA and genetic programming simultaneously. GEP employs the chromosomes and expression trees that are provided as programs. The chromosomes are usually a combination of genes with the same size, and the programs provide coded genetic data in chromosomes [13]. GEP is a phenotype/genotype system whose genotype and phenotype are completely separate from each other. In this method, linear chromosomes and expression trees represent phenotype and genotype, respectively [12]. The process of data decoding from chromosomes to expression trees, known as "translation", consists of a set of rules. These rules are related to the organization of functions and terminals in expression trees, and indicate the connection between different sub-expression trees (Sub-ETs). In order to create chromosomes and genes, the terminal set (TS) and the function set (FS) should be defined. FS consists of various signs such as  $FS = \{+, -, \times, /, \sqrt{\cdot}\}$ . TS is composed of different components that represent different variables and fixed values (for example {a, b, c, 0, 1, 2}). Genes used in GEP contain two types of different information. The first type includes the data that is used to provide the GEP model and is stored in the head, while the second type consists of the terminals that are stored in the tail and employed for generating the next models. The length of genes in GEP is calculated by the following formula:

$$l = h + t = h + h \times (n - 1) + 1 \quad (1)$$

where  $l$  is the gene length,  $h$  is the head,  $t$  is the tail, and  $n$  is the number of arguments in a mathematical function that has the highest number of arguments. Figure 2 illustrates a sample of the organized GEP model.

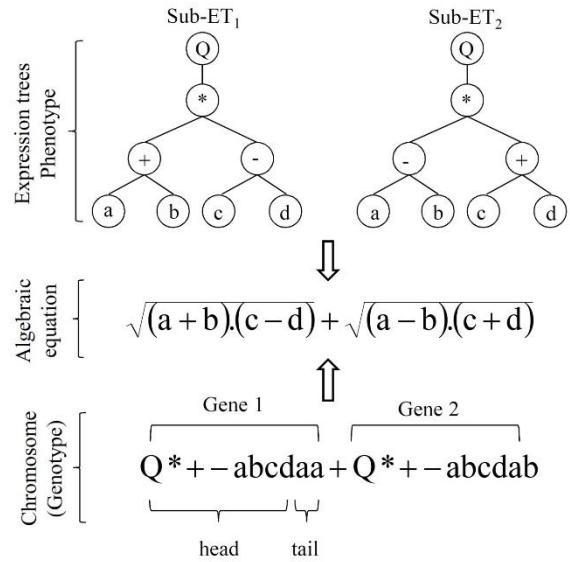


Figure 2. Chromosomes related to a GEP model.

FS should be determined for each non-linear problem (Table 1). FS is the evolutionary nature in the GEP model and so allows unlimited changes in a gene or among different genes in a chromosome. Firstly, GEP creates a random distribution of functions and terminals in the chromosome genes regarding the problem of interest. The initial individuals generated randomly are called "parents". These parents are produced to create an off-spring using the genetic operators. In order to create a new off-spring adapted with the environment and more chance to survive, each individual benefits from its genetic information. In the evolutionary process of a function, the natural selection procedure is based on the fitness of the relationships related to an off-spring producing less error. Hence, GEP benefits from an evolutionary process to reach the best off-spring without an evolution stop in the next generations. If the fitness function used in this study is considered as the root relative squared error (RRSE), the fitness function related to the  $i$ th program is calculated as follows:

$$f_i = \frac{1000}{1 + RRSE_i} \quad (2)$$

By considering the fact that the  $RRSE_i$  value can be zero to infinity, the value of the fitness function is placed on the domain of 0 to 100. The  $RRSE_i$  fitness function is calculated by the following equation:

$$RRSE_j = \sqrt{\frac{\sum_{i=1}^n (p_{ij} - O_i)^2}{\sum_{i=1}^n (O_i - \bar{O})^2}} \quad (3)$$

where,  $P_{ij}$  is the value predicted by the  $j$ th program for the  $i$ th fitness,  $O_i$  is the observed value,  $\bar{O}$  is the average of the observed values, and  $n$  is the number of samples. It should be stated that the key parameters used in the current work were adjusted by a trial-and-error procedure to find the most optimized values. In table 1, the optimized values of the key parameters related to the GEP model were tabulated.

**Table 1. Optimized values of key parameters related to the GEP model.**

Parameters	Setting
P1	Number of generations
P2	250000
P3	Number of chromosomes
P4	200
P5	Number of genes
P6	6
P7	Head size
P8	4
P9	Linking function
P10	Addition
P11	Type of fitness function
P12	RRSE
P13	Mutation rate
P14	0.01
P15	Inversion rate
	0.05
	IS Transposition
	0.05
	Ris Transposition
	0.05
P11	One-point recombination rate
P12	0.35
P13	Two-point recombination rate
P14	0.35
P15	Gene recombination rate
	0.2
	Gene transportation rate
	0.2
	+, -, ×, /, $x^2$ , sqrt,
	Pow, Log, Exp,
	sin, Atan

**Table 2. The minimum, maximum, variance, and standard deviation of the experimental parameters measured by Kumar et al. [19].**

	$Q$	$S_0$	$K_s$	$h_1$	$h_2$	$L_j$
Maximum	0.072	0.016	0.011	0.087	0.344	0.9
Minimum	0.034	0.005	0.002	0.03	0.026	0.3
Average	0.057	0.009	0.006	0.053	0.262	0.638
Variance	$8.98 \times 10^{-5}$	$3.79 \times 10^{-5}$	$9.09 \times 10^{-6}$	0.0002	0.002	0.017
Standard deviation	0.009	0.006	0.003	0.012	0.044	0.129

### 2.3. Hydraulic Jump Length on Sloping Rough Bed

Different researchers such as Hager *et al.* [20], Ead S. & Rajaratnam [1], and Carollo *et al.* [5] have assumed that the length of the hydraulic jump is a function of the parameters such as the flow Froude number at the hydraulic jump upstream ( $F_1$ ) and the ratio of bed roughness to the flow depth at the hydraulic jump upstream ( $K_s/h_1$ ). Also Azimi *et al.* [21-23] have assumed the hydraulic jump length on rough beds as a function of the Froude number ( $F_1$ ), the ratio of bed roughness to the flow depth at the upstream of the hydraulic jump ( $K_s/h_1$ ), and the sequent depth ratio ( $h_2/h_1$ ). Also, Kumar and Lodhi [19], by establishing an experimental research work, have examined the influence of the channel slope ( $S_0$ ).

### 2.2. Experimental Model

It should be stated that natural channels and hydraulic structures own sloping rough beds. In contrast, most of the studies related to the hydraulic jump have been implemented on the smooth bed with trivial slope. Also the length of hydraulic jump is a significant variable to calculate the length of stilling basins or other structures. Therefore, in the current work, the length of hydraulic jump in a sloping rough channel is estimated. To do this, the experimental measurements obtained by Kumar *et al.* [19] are employed. The experimental model is composed of a rectangular channel in which the experiments are conducted in three slopes including 0.000463, 0.00986, and 0.01552. The length and width of the rectangular channel are 8 m and 0.6 m, respectively. The rough bed is created by means of stone materials for four different roughness conditions with the average diameters ( $d_{50}$ ) of 0.00398 m, 0.0056 m, 0.007 m, and 0.011 m. Kumar *et al.* [19] have measured the values for the flow rate ( $Q$ ), bed slope ( $S_0$ ), bed roughness height ( $K_s$ ), and flow depth at the hydraulic jump upstream ( $h_1$ ) and the hydraulic jump length ( $L_j$ ). The maximum, minimum, average, variance, and standard deviation of the experimental parameters are listed in table 2.

Thus in the current work, the influence of the flow Froude number ( $F_1$ ), the ratio of bed roughness ( $K_s/h_1$ ), the sequent depth ratio ( $h_2/h_1$ ), and the channel bed slope ( $S_0$ ) are considered on the length of the hydraulic jump:

$$\frac{L_j}{h_1} = f\left(F_1, \frac{h_2}{h_1}, \frac{K_s}{h_1}, S_0\right) \quad (4)$$

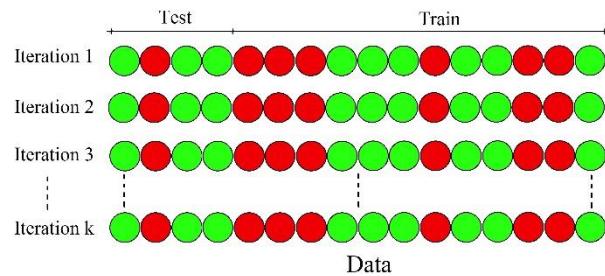
As discussed, for modeling the length of the hydraulic jump using the GEP model, the influence of the Froude number at the hydraulic jump upstream ( $F_1$ ), the ratio of bed roughness ( $K_s/h_1$ ), the sequent depth ratio ( $h_2/h_1$ ), and the bed slope ( $S_0$ ) are considered. Also in this work, the Monte Carlo simulations are used to enhance the capability of the numerical models. These simulations are a broad classification of

computational algorithms using random sampling for calculating the numerical results. The main idea of this method relies on this principle that using random making-decisions solves the phenomena that might be actual in nature. The Monte-Carlo methods are usually implemented for simulating the physical and mathematical systems that are not solvable by means of other methods. Generally, this method solves different problems such as optimization and numerical integration using the probability distribution. In the current work, the Monte Carlo simulation was encoded with the GEP model to increase the number of iterations during the numerical simulation (for instance, 1000 iterations), meaning that in each iteration of the GEP model, the simulation was iterated 1000 times using the Monte Carlo simulation. Therefore, the flexibility of the numerical model was significantly enhanced.

There are some methods available to verify the numerical models, for example, the traditional training and testing or k-fold cross-validation method. In the traditional training and testing method, each observation value is used in just the training or testing mode, whereas all the observed values are utilized in both the training and testing procedures at least once.

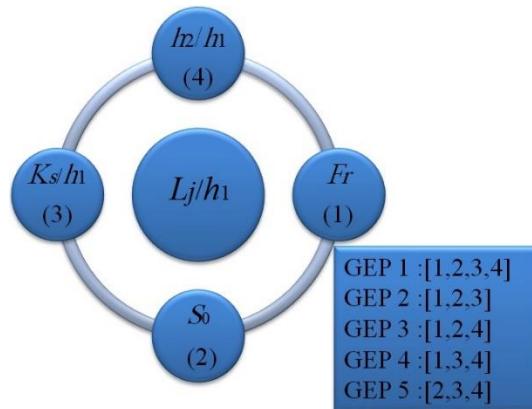
Generally, the k-fold cross-validation method is utilized for validation of the mentioned models. In the k-fold cross-validation method, the main sample is divided into  $k$  sub-samples with the same size randomly. Among  $k$  sub-samples, one sub-sample is used as the validation data and the remaining as the test data for each one of the GEP models. Then the method repeats  $k$  times (equal to the number of layers) so that each  $k$  sub-sample is used exactly once as the validation data once. Then the results obtained from the mentioned  $k$  layers are averaged and provided as an approximation. The advantage of this method is the random repetition of sub-samples in the test and learning process for all observations. In other words, each observation is used exactly once for the numerical model validation. The schematic layout of k-fold cross-validation is shown in figure 3.

In this work, five different GEP models are introduced for modeling the hydraulic jump length occurring on rough sloping bed. The combinations of the input parameters for five GEP models are illustrated in figure 4.



**Figure 3. Schematic layout of the k-fold cross-validation.**

It is worth mentioning that these five GEP models were defined to perform the sensitivity analysis of the input parameters. This means that GEP 1 was a function of all input parameters, and then the effect of each input was eliminated for GEP 2 to GEP 5. After that, the performance of GEP 1 to GEP 5 was compared, and lastly, the levels of effectiveness of different input parameters were easily identified.



**Figure 4. Combinations of the input parameters for simulating objective function of hydraulic jump length for different GEP models.**

### 3. Results and Discussion

#### 3.1. Criteria for Examination of Accuracy of Numerical Models

In the current work, in order to examine the accuracy of different GEP models, the statistical indices including the correlation coefficient ( $R$ ), mean absolute percent error (MAPE), root mean square error (RMSE), scatter index (SI), and BIAS are employed. The best GEP model should have a reasonable accuracy (MAPE, RMSE, SI, and BIAS) and an acceptable correlation with the experimental measurements ( $R$ ). Additionally, the accuracy of the model should be evaluated by means of the absolute (RMSE) and relative criteria (MAPE) indices simultaneously. Therefore, all the statistical indices are required to assess the GEP models appropriately.

$$R = \frac{\sum_{i=1}^n \left( (Lj/h_1)_{(Observed)_i} - \overline{(Lj/h_1)_{(Observed)}} \right) \left( (Lj/h_1)_{(Predicted)_i} - \overline{(Lj/h_1)_{(Predicted)}} \right)}{\sqrt{\sum_{i=1}^n \left( (Lj/h_1)_{(Observed)_i} - \overline{(Lj/h_1)_{(Observed)}} \right)^2 \sum_{i=1}^n \left( (Lj/h_1)_{(Predicted)_i} - \overline{(Lj/h_1)_{(Predicted)}} \right)^2}} \quad (5)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left( \frac{|(Lj/h_1)_{(Predicted)_i} - (Lj/h_1)_{(Observed)_i}|}{(Lj/h_1)_{(Observed)_i}} \right) \times 100 \quad (6)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left( (Lj/h_1)_{(Predicted)_i} - (Lj/h_1)_{(Observed)_i} \right)^2} \quad (7)$$

$$SI = \frac{RMSE}{\overline{(Lj/h_1)_{(Observed)}}} \quad (8)$$

$$BIAS = \frac{1}{n} \sum_{i=1}^n \left( (Lj/h_1)_{(Predicted)_i} - (Lj/h_1)_{(Observed)_i} \right) \quad (9)$$

In Equations (5) to (9),  $(Lj/h_1)_{(Observed)_i}$  is the length of the experimental hydraulic jump,  $(Lj/h_1)_{(Predicted)_i}$  is the length of the predicted hydraulic jump,  $\overline{(Lj/h_1)_{(Observed)_i}}$  is the average of the experimental hydraulic jumps, and n is the number of experimental measurements. The values of R, MAPE, RMSE, SI, and BIAS for different GEP models are shown in table 3. In addition, the scatter plots for the values of the hydraulic jump lengths simulated by the GEP 1 to GEP 5 models are shown in figure 5. As mentioned, the FEP 1 model simulates the hydraulic jump length as a function of all the input parameters. According to the modeling results, among all the defined models, the results of this model have the highest correlation with the experimental values. The R value for GEP 1 was computed to be 0.901. In contrast, the minimum error values were achieved for GEP 1. In other words, the values of MAPE and RMSE for this model were calculated to be 11.517 and 1.664, respectively. However, the BIAS value is equal to -0.021. In this paper, four models with a combination of three input parameters are introduced (GEP 2 to GEP 5 models). The GEP 2 model approximates values of the hydraulic jump length on sloping rough beds in terms of the flow Froude number, the ratio of bed roughness, and the channel slope. In other words, the effects of the sequent depth ratio ( $h_2/h_1$ ) are eliminated. For the mentioned model, the values of SI, RMSE, and BIAS were obtained to be 0.149, 1.872, and -0.023, respectively. Also the value of R for GEP 2 was computed to be 0.873. In the GEP 3 model, the influence of the ratio of the channel bed roughness on the modeling results is neglected. It should be noted that the GEP 3 model is a

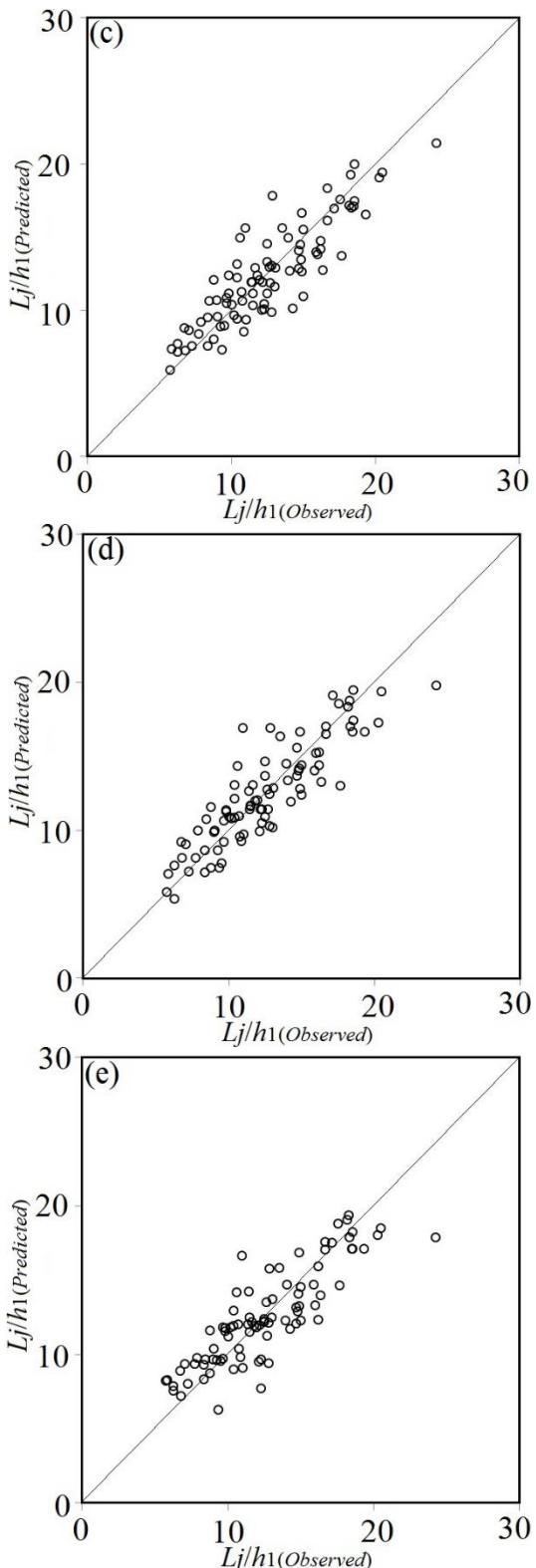
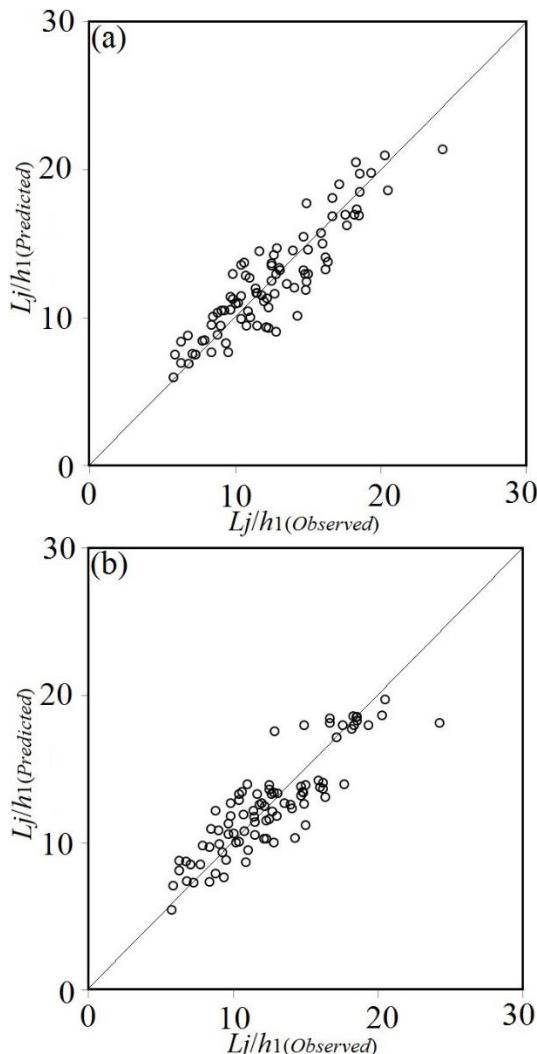
function of the flow Froude number, the sequent depth ratio, and the channel bed slope. For GEP 3, the values of R and SI were estimated to be 0.876 and 0.147, respectively. Furthermore, the values of MAPE, RMSE, and BIAS for the mentioned model were calculated to be 12.459, 1.850, and -0.091, respectively. Among all models with three input parameters, the GEP4 model has a higher accuracy. The R value for this model is equal to 0.879. Also MAPE and RMSE for this model were approximated to be 12.211 and 1.828, respectively. It is worth noting that the GEP 4 model simulates values of the hydraulic jump length in terms of  $F_1, h_2/h_1, Ks/h_1$ . For this model, the effects of the channel bed slope were removed. According to the modeling results, among all the GEP models, GEP 5 has the maximum error and the lowest correlation coefficient. This model simulates values of the hydraulic jump length in terms of the sequent depth ratio, ratio of bed roughness, and channel bed slope. In other words, the effects of the flow Froude number are neglected in this model. For the GEP 5 model, the R value was estimated to be equal to 0.856. Furthermore, the values of MAPE, RMSE, and SI for this model were obtained to be 13.566, 1.987, and 0.158, respectively. Thus according to the results of different GEP models, GEP 1 was detected as the superior model. In addition, by eliminating the Froude number at the hydraulic jump upstream, the modeling accuracy is reduced dramatically. Therefore, the mentioned parameter is identified as the most effective input parameter in estimating the hydraulic jump length on sloping rough beds.

Regarding the simulation results, GEP 1 was identified as the best GEP model. Also GEP 4, GEP 5, and GEP 3 were detected as the second,

third, and fourth best GEP models. Moreover, GEP 2 had the lowest accuracy and worst performance to estimate the target function. According to the performed sensitivity analysis, the Froude number owned the highest effectiveness on modeling the roller length of hydraulic jump. Furthermore, the ratio of bed roughness to the flow depth ( $K_s/h_1$ ), sequent depth ratio ( $h_2/h_1$ ), and the channel bed slope ( $S_0$ ) were introduced as the most influencing input parameters.

**Table 3.** Values of different statistical indices for GEP models.

	<i>R</i>	MAPE	RMSE	SI	BIAS
GEP 1	0.901	11.517	1.664	0.133	-0.021
GEP 2	0.873	12.748	1.872	0.149	-0.023
GEP 3	0.876	12.459	1.850	0.147	-0.091
GEP 4	0.879	12.211	1.828	0.146	-0.068
GEP 5	0.879	13.566	1.987	0.158	-0.032



**Figure 5.** Comparison of hydraulic jump length measured and predicted by a) GEP 1 (b) GEP 2 (c) GEP 3 (d) GEP 4 (e) GEP 5.

Regarding the simulation results, GEP 1 was identified as the best GEP model. Also GEP 4, GEP 5, and GEP 3 were detected as the second, third, and fourth best GEP models. Moreover, GEP 2 had the lowest accuracy and worst performance to estimate the target function.

According to the performed sensitivity analysis, the Froude number owned the highest effectiveness on modeling the roller length of hydraulic jump. Furthermore, the ratio of bed roughness to the flow depth ( $K_s/h_1$ ), sequent depth ratio ( $h_2/h_1$ ), and the channel bed slope ( $S_0$ ) were introduced as the most influencing input parameters.

### 3.2. Partial Derivative Sensitivity Analysis (PDSA)

By introducing GEP 1 as the superior model, a relationship was provided for calculating the length of hydraulic jumps on sloping rough beds. This formula estimates values of the hydraulic jump length ( $L_j/h_1$ ) as a function of the Froude number ( $F_1$ ), the ratio of bed roughness ( $K_s/h_1$ ), the sequent depth ratio ( $h_2/h_1$ ), and the bed slope ( $S_0$ ).

Generally, PSDA is performed for identifying the influence of the input parameters on the objective parameter. In other words, PSDA is a method for identifying the changing pattern of the objective parameter according to the input parameters. A positive PSDA means that the objective function (the roller length) is increasing, while a negative

sign means that the objective function is decreasing. In other words, in this method, the relative derivative of each input parameter is calculated according to the objective function.

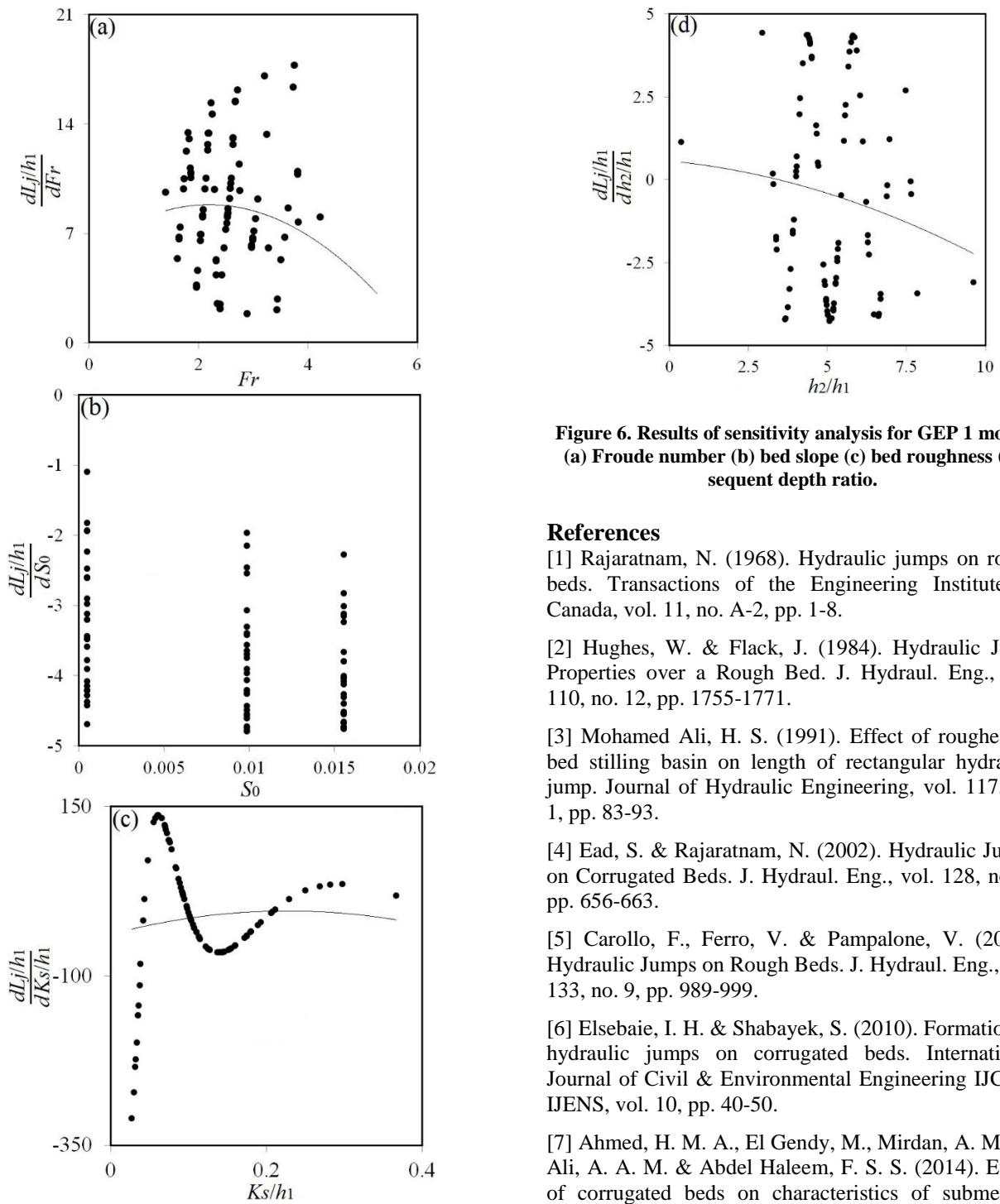
In the followings, the partial derivative sensitivity analysis (PDSA) is carried out to examine the effects of the input variables on the output of the GEP 1 model. The positive sign of the partial derivative means that by increasing the input parameter, the output value increases. Figure 5 illustrates the results of the sensitivity analysis for the GEP 1 input parameters. As shown, for all Froude number ( $F_1$ ) values, the sensitivity analysis was calculated to be positive, thus by increasing the Froude number value, the hydraulic jump values increased as well. Furthermore, for all values of the bed slope ( $S_0$ ), the sensitivity analysis value was obtained to be negative. It means that the output parameter (hydraulic jump length) has decreased. According to figure 6, the behavior of the parameters  $K_s/h_1$  and  $h_2/h_1$  versus the sensitivity analysis is complex, and a part of the sensitivity analysis results was calculated to be positive and negative for another part.

$$\frac{L_j}{h_1} = \left( F_1 \right)^{\frac{4\sqrt{F_1}}{F_1 + S_0}} + F_1^{(42.54 \times \sqrt{F_1})} + \sin \left( -8.14 \times \log \left( \frac{K_s}{h_1} \times \exp \left( \frac{K_s}{h_1} \right) \right) \right) + \sin \left( -1.41 \times \frac{h_2}{h_1} \right) \tan \left( 6.14 \times \frac{h_2}{h_1} \right) \quad (10)$$

## 4. Conclusions

Determination of the hydraulic jump length for estimating the length of detention ponds is very important. In this paper, using the gene expression programming (GEP), the length of the hydraulic jump occurring on sloping rough beds was modeled. To this end, five different GEP models were defined according to the parameters affecting the hydraulic jump length. Then by analyzing the results of the mentioned models, the superior model and the most effective parameter were detected. The superior model predicts the values of the hydraulic jump length with a reasonable accuracy. For example, the values of  $R$ ,  $MAPE$ , and  $RMSE$  for the mentioned model

were obtained to be 0.901, 11.517, and 1.6644, respectively. Also the flow Froude number was considered as the most effective parameter in estimating the hydraulic jump length. Then a relationship was proposed for calculating the hydraulic jump length on sloping rough beds. Furthermore, by conducting a sensitivity analysis, it was concluded that by increasing the Froude number value, the hydraulic jump length also increased. According to PDSA, PDSA increased by increasing the Froude number parameter. It is suggested that the optimization algorithms like genetic algorithm (GA) or particle swarm optimization (PSO) can be used to enhance the performance of the GEP model.



**Figure 6. Results of sensitivity analysis for GEP 1 model**  
 (a) Froude number (b) bed slope (c) bed roughness (d) sequent depth ratio.

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## مدل سازی طول پرش هیدرولیکی بر روی بستر زبر شیب دار با استفاده از برنامه نویسی بیان ژن

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## چکیده:

معمولًا جریان در پائین دست سریزهای اوجی جریان فوق بحرانی دارای انژی فراوانی بوده که برای محافظت از سازه ها و تاسیسات پائین دست، حوضچه های آرامش تعییه می شوند. به منظور طراحی بهینه حوضچه های آرامش، تعیین طول پرش هیدرولیکی از اهمیت قابل توجهی برخوردار است. در مطالعه حاضر، با استفاده از مدل برنامه نویسی بیان ژن (GEP) طول پرش های هیدرولیکی بر روی بستر زبر شیب دار مدل سازی شد. مدل برنامه نویسی بیان ژن یک روش تکاملی است که توانایی توسعه برنامه های کامپیوترا پیچیده را دارد. روند حل مدل برنامه نویسی بیان ژن بدین گونه است که تعدادی از برنامه های کامپیوترا مختلف تکامل می بایند و تابع برآش جهت حل مسئله مورد نظر استفاده می شود. در مطالعه حاضر، برای بررسی توانایی مدل های عددی از شبیه سازی های مونت کارلو استفاده شد. در مقابل، از روش اعتبار سنجی چند لایه ای برای بررسی توانایی مدل های GEP استفاده گردید. به منظور تعیین طول پرش هیدرولیکی با استفاده از پارامترهای ورودی پنج مدل مختلف GEP معرفی شد. سپس با تجزیه و تحلیل نتایج مدل های GEP، مدل برتر معرفی شد. مدل مذکور مقادیر طول پرش هیدرولیکی را با دقت مناسبی تخمین می زند. برای مدل برتر مقادیر ضریب همبستگی، درصد میانگین مطلق خطای جذر میانگین مربعات به ترتیب برابر  $0.901$ ،  $0.517$  و  $0.664$  محسوبه شد. همچنین عدد فروند در بالادست پرش هیدرولیکی به عنوان موثرترین پارامتر در مدل سازی طول پرش هیدرولیکی شناسایی شد. در ادامه یک رابطه برای محاسبه طول پرش هیدرولیکی بر روی بستر زبر شیب دار ارائه گردید. این رابطه مقادیر طول پرش را بر حسب عدد فروند جریان، نسبت زبری بستر، نسبت اعماق مزدوج و شیب بستر محاسبه می کند. همچنین آنالیز حساسیت نشان داد که با افزایش عدد فروند جریان مقادیر طول پرش هیدرولیکی افزایش یافت.

کلمات کلیدی: طول پرش هیدرولیکی، بستر زبر شیب دار، آنالیز حساسیت، برنامه نویسی بیان ژن.