Journal of AI and Data Mining
Vol 6, No 2, 2018, 365-373
DOI: 10.22044/JADM.2017.4670.1553

Non-linear Fractional-Order Chaotic Systems Identification with Approximated Fractional-Order Derivative based on a Hybrid Particle Swarm Optimization-Genetic Algorithm Method

M. kosari[1*] and M. Teshnehlab[2]

1. Electrical Engineering-Control Department, K.N.Toosi University of Technology, Tehran, Iran.
2. Faculty of Electrical Engineering-Control Department, K.N.Toosi University of Technology, Tehran, Iran.

Received 30 August 2016; Revised 12 December 2016; Accepted 19 July 2017
**Corresponding author: mkosari@mail.kntu.ac.ir (*M. kosari*).*

Abstract
Although many mathematicians have searched on the fractional calculus since many years ago, its application in engineering, especially in modeling and control, does not have many antecedents. Since there is much freedom in choosing the order of differentiator and integrator in fractional calculus, it is possible to model the physical systems accurately. This paper deals with the time-domain identification fractional-order chaotic systems, where conventional derivation is replaced by a fractional one with the help of a non-integer derivation. This operator is itself approximated by an *N*-dimensional system composed of an integrator and a phase-lead filter. A hybrid particle swarm optimization (PSO)-genetic algorithm (GA) method is applied to estimate the parameters of the approximated non-linear fractional-order chaotic system modeled by a state-space representation. The feasibility of this approach is demonstrated through identifying the parameters of the approximated fractional-order Lorenz chaotic system. The performance of the proposed algorithm is compared with GA and standard particle swarm optimization (SPSO) in terms of parameter accuracy and cost function. In order to evaluate the identification accuracy, the time-domain output error is designed as the fitness function for parameter optimization. The simulation results show that the proposed method is more successful than the other algorithms for parameter identification of the fractional-order chaotic systems.

Keywords: *Parameter Identification, Chaotic System, Particle Swarm Optimization, Genetic Algorithm, Fractional Calculus.*

1. Introduction

In the last two decades, fractional calculus has been applied in an increasing number of fields due to the fact that the fractional-order modeling can describe the real-world physical phenomena more reasonably and accurately than the classical integer order calculus [1, 2]. It has been found that many real objects and processes in interdisciplinary fields can be described by fractional differential equations.

The diffusion of heat into a semi-infinite solid [2], voltage-current relation of a semi-infinite lossy transmission line [3], viscoelastic systems [4], dielectric polarization [5], model of love between humans [6], model of happiness [7], and model of non-local epidemics [8] are just a few examples of the fractional calculus applications.

Different methods have been proposed for the identification of fractional-order systems. Most of them consist of the generalization to fractional order systems of standard methods that are used in the identification of systems with integer order derivatives.

Although many methods have been proposed for parameter identification of integer-order chaotic systems [9-13], a little work has been done for fractional-order chaotic systems [14-16].

Time domain methods have been introduced, for example in [17, 18], where a method based on the discretization of a fractional differential equation using Grunwald-Letnikov's definition has been introduced, and the parameters have been estimated using the least-squares approach. In [19], a method based on the approximation of a

fractional integrator by a rational model has been proposed. In [20], the use of methods based on fractional orthogonal bases has been introduced. In [21], identification of fractional-order systems using the modulating function method in case of noisy measurements has been proposed. Other techniques can also be found, for example in [22, 23].

In this paper, we deal with time domain identification of fractional-order non-linear systems modeled by a state-space representation. The general problem is the identification of a non-linear fractional-order system, and particularly, estimation of the fractional derivation order of fractional-order.

According to the approach proposed in [19], we intended to describe the input–output behavior of the system. Estimation of parameters of the model represents a non-linear problem that we proposed to solve using the PSO-GA method. The first problem to solve was that of numerical integration of the fractional differential system. In other words, it is necessary to propose a fractional derivation operator in order to simulate the system in a conventional way. This operator is defined using the frequency considerations derived from the approach initiated by Oustaloup [19]. The particularity of this approach is to rely on the numerical simulation of the model in order to generate the output error, and on the minimization of the resulting quadratic criterion using a PSO-GA method.

In the next section, basic definitions of fractional derivatives, integrals, and non-linear fractional-order systems are recalled. The output error method based on the PSO-GA method is applied to the identification of fractional-order non-linear chaotic system in Section III. The numerical results are presented in Section IV, followed by conclusions, summarizing the main results obtained.

2. Mathematical background

2.1. Fractional non-linear systems

Fractional-order calculus is the generalization of the classical integer order calculus. In this paper, we will consider the general incommensurate fractional-order non-linear system, represented as follows [1]:

$$\begin{aligned} &{}_0D_t^{\alpha_i} x_i(t) = f_i(x_1(t), x_2(t), \cdots, x_n(t), t), \\ &x_i(0) = c_i,\ i = 1, 2, \cdots, n, \\ &y(t) = C\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T \end{aligned} \tag{1}$$

where, c_i is the initial conditions, α_i is the fractional orders, and $C = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}$.

The equilibrium points of system (1) are calculated via solving the following equation:

$$f(X) = 0 \tag{2}$$

and we suppose that $E^* = (x_1^*, x_2^*, \cdots, x_n^*)$ is an equilibrium point of system (1).

2.2. Fractional derivative

There are several definitions for fractional-order derivative [2], three most commonly used ones are the Grunwald-Letnikov, Riemann-Liouville, and Caputo derivation definitions.

The Riemann-Liouville derivative definition of the order α can be described as:

$${}_c^R D_t^{\alpha} f(t) = \frac{d^{\alpha}}{dt^{\alpha}}\left[\frac{1}{\Gamma(n-\alpha)}\int_c^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau\right], \tag{3}$$

where, $n-1 \le \alpha < n, n \in N$ and $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx$ is the gamma function.

The Caputo derivative definition has the following form:

$${}_c^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_c^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau, \tag{4}$$

The Grunwald-Letnikov's derivation definition can be written as:

$${}_c^{GL} D_t^{\alpha} f(t)\big|_{t=kh} = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left\lfloor \frac{t-c}{h} \right\rfloor} \omega_j^{(\alpha)} f(kh-jh), \tag{3}$$

where, h is the sample time, $\lfloor . \rfloor$ is the flooring function, and the coefficient

$$\omega_j^{(\alpha)} = \frac{(-1)^j \Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)},\ j = 0, 1, \cdots. \tag{4}$$

2.3. Fractional integration

The αth non-integer order Riemann–Liouville integral (α real positive) of a function $f(t)$ can be defined by the following relation [1]:

$$I_{\alpha}(f(t)) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau \tag{5}$$

2.4. Approximation of fractional operators

Fractional operators are usually approximated by high order rational models. As a result, a fractional model and its rational approximation have the same dynamics within a limited frequency band. The most commonly used approximation of s^{α} in the frequency band $[\omega_b, \omega_h]$ is the recursive distribution of zeros and poles proposed by Oustaloup [17]. Trigeassou et

al. [24] have suggested to use an integrator outside the frequency range $[\omega_b, \omega_h]$ instead of a gain:

$$I_\alpha(s)=\frac{1}{s^\alpha},$$
$$I_\alpha^*(s)=\frac{C_0}{s}\left(\frac{1+s/\omega_b}{1+s/\omega_h}\right)^{1-\alpha} \approx \frac{C_0}{s}\prod_{k=1}^{N}\frac{1+s/\omega'_k}{1+s/\omega_k} \quad (6)$$

The block diagram of approximation of fractional integration, relation (8), can be represented as in figure 1:

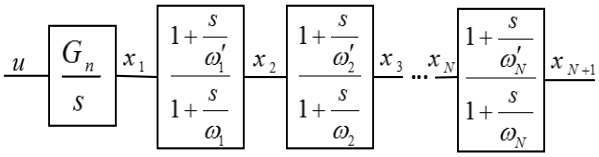

Figure 1 $I_\alpha^*(s)$ Block diagram [19].

The operator $I_\alpha^*(s)$ is characterized by six parameters, where ω'_1 and ω_N define the frequency range, N is the number of cells (it is directly related to the quality of the desired approximation), and pulsations ω_i and ω'_i are linked by:

$$\omega_i=\lambda\omega'_i, \quad \text{with} \quad \lambda>1$$
$$\omega'_{i+1}=\eta\omega_i, \quad \text{with} \quad \eta>1 \quad (7)$$

The fractional order of operator is:

$$\alpha=1-\frac{\log(\lambda)}{\log(\lambda\eta)} \quad (8)$$

where, λ and η are real parameters that depend on the differentiation order α. A bigger N causes a better approximation of the integrator $I_\alpha(s)$.

As $I_\alpha^*(s)$ is composed of a product of cells, we define the state-variables as the output of each cell [19], according to figure 1.

This system corresponds to the state-space representation:

$$M\dot{x}=Ax+Bu$$

or equivalently:

$$\dot{x}=A^*x+B^*u,$$
$$A^*=M^{-1}A, \quad B^*=M^{-1}B. \quad (9)$$

where

$$M=\begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -\lambda & 1 & 0 & \dots & 0 \\ 0 & -\lambda & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & 0 \\ 0 & \dots & 0 & -\lambda & 1 \end{bmatrix},$$

$$A=\begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ \omega_1 & -\omega_1 & 0 & \dots & 0 \\ 0 & \omega_2 & -\omega_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \omega_N & -\omega_N \end{bmatrix},$$

$$B=\begin{bmatrix} G_n \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, \quad x=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_{N+1} \end{bmatrix}.$$

Thus fractional integrator operator corresponds to the symbolic representation given in figure 2.

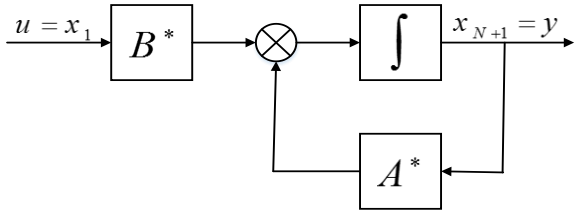

Figure 2. State-space representation of operator [19].

3. Fractional identification technique

According to figure 2, for using this approximation, the non-linear model could be approached by a state-space representation, as in figure 3:

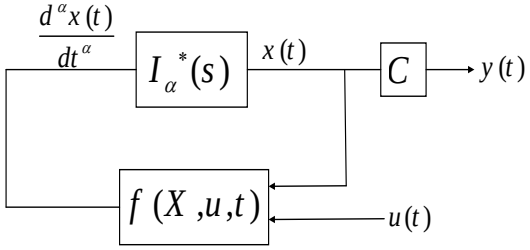

Figure 3. State-space representation of non-linear system.

Thus the approximated state space representation is:

$$\dot{X}(t)=g(X(t),u(t),t,\theta),$$
$$y(t)=CX(t) \quad (10)$$

where, θ consists of the model parameters and operator parameters (ω'_1, ω_N, and α), $C=[0 \ \cdots \ 0 \ 1]$, and:

$$g(X(t),u(t),t,\theta)=A^*X(t)$$
$$+B^*f(X(t),u(t),t). \quad (11)$$

3.1. Output error method

We consider identification and parameter estimation of the non-linear model in the time

domain with the help of M data pairs $\{u_k, y_k^*\}$, where y_k^* represents values of true output.

The model coefficients are conventional parameters that are easily estimated. On the other hand, the real difficulty of the problem deals with the parameter α, which corresponds to a complex reality. Thus the identification problem is non-linear, and it is justified to use an output-error technique.

Let us consider the quadratic criterion:

$$J = \frac{1}{M}\sum_{k=1}^{M}(y_k^* - \hat{y}(u,\hat{\theta}))^2 \qquad (12)$$

where, $\hat{y}_k(u,\hat{\theta})$ represents the numerical simulation of non-linear model based on estimation $\hat{\theta}$, and excited by input $u(t)$.

$\hat{y}(t)$ is non-linear in the parameters $\hat{\theta}$; thus quadratic criterion J has to be minimized iteratively by an optimization algorithm, while obtaining proper parameters, $\hat{\theta}$. In the case of linear or non-linear rational systems, the numerical simulation of the model is a very classical and simple problem. This technique [19] can be schematized by the drawing in figure 4.

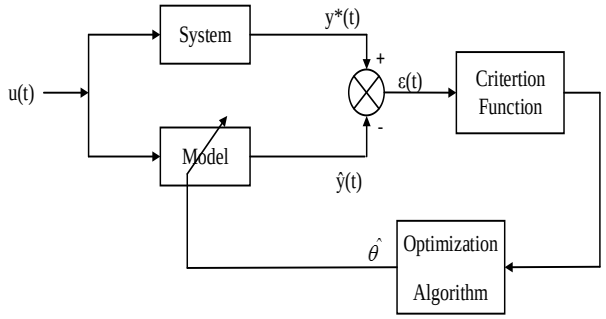

Figure 4. Output-error identification technique.

3.2. Genetic algorithm

The genetic algorithm (GA) is a kind of intelligent optimization method, proposed by Holland [25], which imitates the natural genetic phenomena of selection, cross-over and mutation operations in nature. Jong has carried out a great quantity of optimization tests with numerical functions using the Holland's theory, which proves that GA is an effective and efficient stochastic search method [26]. In the 1990s, GA was widely used in the scope of engineering such as reservoir operation optimization [27], numerical model parameter optimization [28], inverse problem research [29], vehicle routing problem [30], and routing in wireless sensor network [31].

GA starts with an initial population that contains a number of individuals, and then new individuals are produced to be better adapted to the environment with random selection, cross-over, and mutation. The best individual is eventually achieved by a number of evolution steps (generations). Every individual in a population is a feasible solution for the optimization problems, and the best individual is the optimal solution to the optimization problem. Compared with the other optimization algorithms, GA has the following advantages:

1. The optimization objective function can be either a continuous function or a discrete function [28];
2. It has the property of global search and automatic convergence to the optimal solution;
3. It is robust in dealing with complex non-linear problems;
4. The principle is simple, easy to understand, versatile, and highly maneuverable.
5. Calculation of sensitivity functions in gradient-based methods is not required.

Many improvements have been made to GA considering its wide applications including a niche technology of cross-over operation [24], a uniform mutation operation method [25], an adaptive algorithm of cross-over and mutation probability [26-27], and the hybrid particle swarm optimization (PSO) and GA method [32]. In this paper, the hybrid PSO-GA method has been used.

3.3. Particle swarm optimization

Standard particle swarm optimization (SPSO) is a kind of swarm intelligence method achieved by individual particle improvements together with cooperation and competition among the whole population [33-35]. The algorithm works by initializing a flock of birds randomly over the searching space, where every bird is called as a "particle". These "particles" fly with a certain velocity and find the global best position after some iteration. At each iteration, each particle can adjust its velocity vector based on its momentum and the influence of its best position as well as the best position of the best individual. Then the particle flies to a newly computed position. Suppose that the search space is *n*-dimensional, and then the position and velocity of the ith particle are represented by $x_i = [x_{i1}\ x_{i2}\ \ldots\ x_{in}]^T$ and $v_i = [v_{i1}\ v_{i2}\ \ldots\ v_{in}]^T$, respectively. The fitness of each particle can be evaluated according to the objective function of the optimization problem. The best previously visited position of the particle *i* is noted as its personal best position,

denoted by $p_i = [p_{i1}\ p_{i2}\ \dots\ p_{in}]^T$. The position of the best individual of the swarm is noted as the global best position $G = [g_1\ g_2\ \dots\ g_n]^T$. At each step, the velocity of a particle and its new position are assigned as follow:

$$v_i(t+1) = \omega\, v_i(t) + c_1 r_1 (p_i - x_i) + c_2 r_2 (G - x_i) \quad (13)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (14)$$

where, t is the current step number, ω is the inertia weight, c_1 and c_2 are the acceleration constants, r_1 and r_2 are two random numbers in the range $[0,1]$, $x_i(t)$ is the current position of the particle, P_i is the best one of the solutions this particle has reached, and G is the best one of the solutions all the particles have reached. The PSO algorithm performs repeated operations of the update equations above until a stopping criterion is reached. In [34], the authors have introduced a constriction factor χ into the PSO algorithm. The aim is to prevent particle explosion and to control convergence. In the PSO algorithm with a constriction factor, the velocity updating process (16) is modified to:

$$v_i(t+1) = \chi [v_i(t) + c_1 r_1 (p_i - x_i) + c_2 r_2 (G - x_i)] \quad (15)$$

where, $c_1 = c_2 = 2.05$ and $\chi = 0.7298$ are the near optimal values in this PSO algorithm. With the new velocity updating (18), the PSO algorithm with a constriction factor is shown to outperform the basic PSO algorithm [34].

ω is the inertia weight that is employed to control the impact of the previous history of velocities on the current velocity. The Linear time-varying weighting function is usually utilized in the following:

$$\omega_{iter} = \frac{(iter_{max} - iter)}{iter_{max}} (\omega_{min} - \omega_{max}) + \omega_{max} \quad (16)$$

where, ω_{iter} is the current weight, ω_{max} is the initial weight, ω_{min} is the final weight, $iter_{max}$ is the maximum iteration number, and $iter$ is the current iteration number [36, 37].

In [32], modifications have been made in PSO using GA to improve the performance and reach global maxima. The genetic operators can be used to prevent premature convergence. Using the cross-over operation, information can be exchanged between two particles that improve the likelihood of searching for the global optimum. Similarly, by applying mutation to PSO, population diversity can be managed.

Hence, a non-linear fractional identification model based on the PSO-GA method can be established through the following steps:

Step 1: Initialization: Set the counter of evolution $t=0$, randomly generate the initial positions $X(0)$ and velocities $V(0)$ and the maximum number of generation $T_m = 500$ as the termination condition.

Step 2: Individual evaluation: Calculate the fitness value of each individual in population $X(t)$.

Step 3: Obtain the new velocities $V(t+1)$ and positions $X(t+1)$ of particles using equations (17) and (18), and then update P_j and G.

Step 4: Selection operation: Apply the tournament selection operation [38] to the population.

Step 5: Cross-over operation: Apply the arithmetic cross-over operation [38] to the population.

Step 6: Mutation operation: Apply the mutation operation [38] to the population. After Steps 4, 5, and 6, new generation population will be obtained, and then update P_j and G again.

Step 7: Termination condition judgment: If $t=T_m$ as the termination conditions, the individual that has the most suitable fitness value in the processing will be selected as the optimal solution; otherwise, back to Step 2. With this, an identification model of non-linear fractional order system is established based on PSO-GA. A working flow chart of the model is shown in figure 5.

5. Simulation results

To demonstrate the effectiveness of the proposed parameter identification method for fractional-order chaotic system, simulation of the fractional-order chaotic Lorenz system is presented. All the algorithms are implemented using the MATLAB 8.1 programming language. In the simulation, the control parameters of all algorithms are set as follow: the population size $M = 20$, the initial inertia weight ω is set to χ, $c_1 = c_2 = 2.05$, the cross-over and mutation probability is equal to 0.7, and the algorithm terminates when a maximum generation number is reached; it is set to 500 generations, and the fitness function is calculated by relation (15).

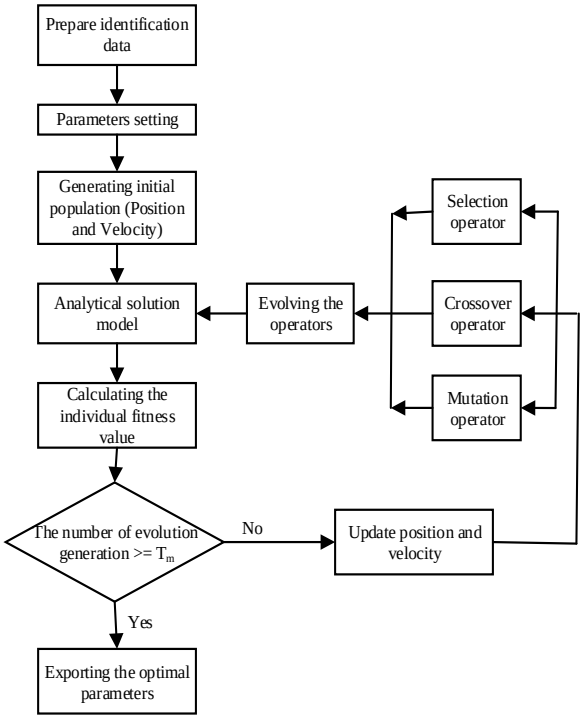

Figure 5. Flow chart for identification model parameters based on PSO-GA.

Now consider the fractional-order commensurate-order Lorenz system [39], described by:

$$\begin{aligned} {}_0D_t^q x(t) &= \sigma(y(t) - x(t)), \\ {}_0D_t^q y(t) &= x(t)(\rho - z(t)) - y(t), \\ {}_0D_t^q z(t) &= x(t)y(t) - \beta z(t), \end{aligned} \qquad (17)$$

where, σ, ρ, β, and q are unknown parameters to be identified.

In this paper, we let the true parameters of system (19) to be $(\sigma, \rho, \beta) = (10, 28, 8/3)$ and $q=0.99$. To obtain the standard state variables x, y, and z, we solve system (20) using the numerical algorithm derived from the G–L definition of fractional derivatives, where the initial condition $(x_0, y_0, z_0)=(0.1,0.1,0.1)$ and the step size $h = 0.001$. The numerical results show that it is chaotic, and its chaotic behavior is shown in figure 6. Then a frequency interval equal to 4 decades is used to approximate the fractional-order derivative with $\omega_b = 10^{-3}$ rad/s and $\omega_h = 10^{1}$ rad/s. Using this approximation and the same initial conditions for the step size, the numerical results for 6 cells is shown in figure 7. As it is seen, both shapes have a chaotic behavior and almost the same action. In this example, the parameters to be identified are σ, ρ, β, and q. In the experiments, the search ranges of parameters are set as $0 \le \sigma \le 20$, $0 \le \rho \le 50$, $0 \le \beta \le 5$, $0 \le \omega_b \le 1$, $0 \le \omega_h \le 100$, and $0 \le q \le 1$.

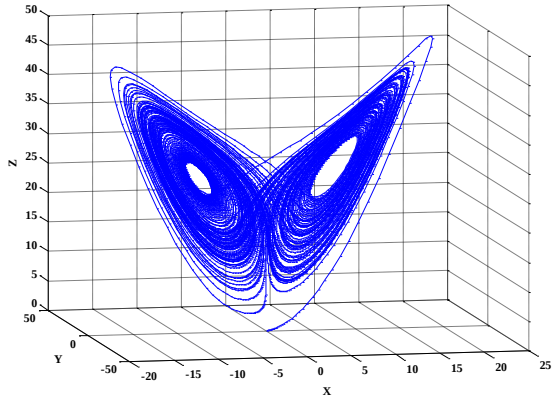

Figure 6. Chaotic behavior of fractional-order Lorenz system with G–L definition of fractional derivatives when $(\sigma, \rho, \beta) = (10, 28, 8/3)$, $q = 0.99$.

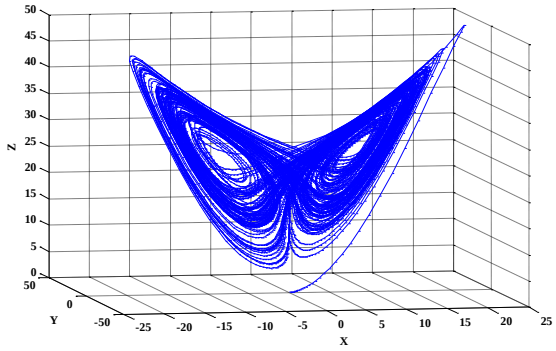

Figure 7. Chaotic behavior of fractional-order Lorenz system with approximated fractional derivatives when $(\sigma, \rho, \beta) = (10, 28, 8/3)$, $q = 0.99$, and $N = 6$.

Tables 1, 2, and 3 list the best results and standard deviations of parameters obtained by algorithms, where each algorithm runs 10 times independently.

From table 1, it can be seen that the best results obtained by GA are different from the true parameter values. According to table 2, the PSO algorithm has achieved better results than the GA method. It can be concluded that the performance of PSO is better than GA. Table 3 represents the best results using the hybrid PSO-GA method, which has derived true parameters with increase in the number of cells. Based on the results of the three tables, the hybrid GA-PSO method was found to be more effective than the other algorithms in terms of the convergence speed and preventing the premature convergence to reach the optimal state.

Table 1. Results obtained by GA method for parameter estimation of approximated fractional-order Lorenz system with 2, 3, 4, 5, and 6 cells.

Cell number (N)	σ	Std of σ	β	Std of β	ρ	Std of ρ	ω_b	Std of ω_b	ω_h	Std of ω_h	q	Std of q	Best J
0(exact)	10	-	28	-	8/3	-	0.001	-	10	-	0.99	-	-
2	8.41	3.02	27.1	4.94	2.36	0.86	0.046	0.032	10.59	6.34	0.9727	0.077	3.267
3	19.88	2.11	27.98	0.262	2.663	0.207	0.026	0.029	9.07	2.277	0.9829	0.012	0.455
4	12.9	6.42	28.07	0.273	2.63	0.555	0.025	0.032	18.11	6.151	0.9813	0.076	0.776
5	16.54	5.61	27.2	0.974	3.29	0.665	0.052	0.027	20	7.668	0.9906	0.223	3.386
6	10.98	4.26	28.03	0.844	3.109	0.572	$6.3e^{-4}$	0.003	2.67	1.391	1	0.087	1.026

Table 2. Results obtained by PSO method for parameter estimation of approximated fractional-order Lorenz system with 2, 3, 4, 5, and 6 cells.

Cell Number (N)	σ	Std of σ	β	Std of β	ρ	Std of ρ	ω_b	Std of ω_b	ω_h	Std of ω_h	q	Std of q	Best J
0(exact)	10	-	28	-	8/3	-	0.001	-	10	-	0.99	-	-
2	9.19	6.47	26.9	6.16	2.51	0.762	0.076	0.028	10.69	5.44	0.9751	0.25	1.387
3	12.93	4.91	28.004	0.035	2.663	0.005	0.002	0.002	10.08	0.013	0.989	0.0014	0.0077
4	10.009	1.87	28.000	0.329	2.6673	0.099	$9.9e^{-4}$	0.002	10.015	3.837	0.9900	0.041	0.0038
5	10.227	3.57	27.992	0.976	2.6778	0.452	$9.6e^{-4}$	0.002	9.827	3.898	0.9903	0.145	0.0347
6	6.32	3.34	27.924	1.39	2.74	0.525	0.0014	0.0005	3.88	4.466	0.9907	0.008	0.5814

Table 3. Results obtained by PSO-GA method for parameter estimation of approximated fractional-order Lorenz system with 2, 3, 4, 5 and 6 cells.

Cell number (N)	σ	Std of σ	β	Std of β	ρ	Std of ρ	ω_b	Std of ω_b	ω_h	Std of ω_h	q	Std of q	Best J
0(exact)	10	-	28	-	8/3	-	0.001	-	10	-	0.99	-	-
2	9.965	2.53	27.987	5.55	2.660	0.822	0.058	1.084	10.031	3.424	0.9819	0.1167	0.0468
3	11.324	4.16	28.001	8.384	2.6659	0.109	0.0014	0.0017	10.026	2.973	0.9896	0.0005	0.0033
4	10.000	4.12	28.000	0.050	2.6667	0.297	0.001	0.0033	10.000	8.375	0.9900	0.058	$2.8e^{-11}$
5	10.000	2.97	28.000	0.189	2.6667	0.227	0.001	0.0000	10.000	2.480	0.9900	0.0277	$2.36e^{-9}$
6	10.000	4.32	28.000	0.488	2.6667	0.520	0.001	0.0019	10.000	6.611	0.9900	0.0418	$9.7e^{-10}$

Also it could be seen that an increase in the number of cells improved the results but caused to get more complicated equations, and it was more difficult to achieve the optimum point. Hence, using 4 cells suggested better results. Figure 8 represents the objective function value (*J*) for algorithms with 4 cells. It shows that the hybrid GA-PSO method is better than the GA and PSO methods in terms of the convergence speed and preventing the premature convergence to reach the optimal point.

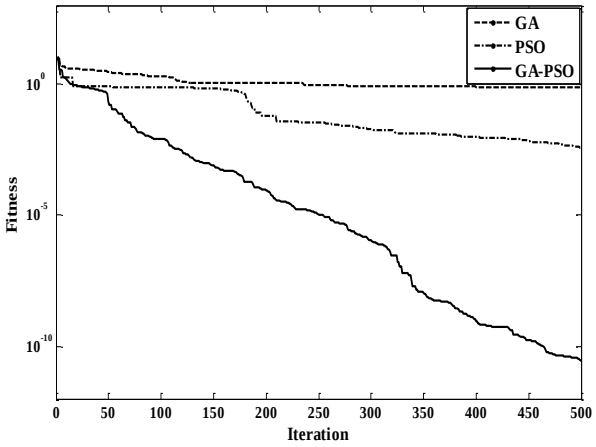

Figure 1. Objective function values J for GA, PSO, and GA-PSO methods.

6. Conclusion

This paper proposes an identification algorithm based on the hybrid PSO-GA method in the time domain using the output error technique for the approximated non-linear fractional-order chaotic systems. The results obtained verify that this algorithm can precisely identify the coefficients and fractional-order of the Lorenz chaotic system. Taking the effective fitness function, the PSO-GA method can do the global search and solve the parameter identification issue for non-linear to the kind of system. Effectively, with the fractional integrator operator, it is possible to consider more complex systems, and then to identify them using the proposed algorithm.

References

[1] Diethelm, K. (2010). The Analysis of Fractional Differential Equations. Springer, Berlin.

[2] Podlubny, I. (1999). Fractional Differential Equations. Academic Press, San Diego.

[3] Wang, J. (1987). Realizations of generalized Warburg impedance with RC ladder networks and transmission lines. Journal of the Electrochemical Society, vol. 134, no. 8, pp. 1915-1920.

[4] Bagley, R. L. & Calico R. A. (1991). Fractional order state equations for the control of viscoelasticallydamped structures. Journal of Guidance, Control, and Dynamics, vol. 14, no. 2, pp. 304-311.

[5] Sun, H., Abdelwahab, A. & Onaral, B. (1984). Linear approximation of transfer function with a pole of fractional power. IEEE Transactions on Automatic Control, vol. 29, no. 5, 29(5), pp. 441-444.

[6] Ahmad, W. M. & El-Khazali, R. (2007). Fractional-order dynamical models of love. Chaos, Solitons & Fractals. vol. 33, no. 4, pp. 1367-1375.

[7] Song, L., Xu, S. & Yang, J. (2010). Dynamical models of happiness with fractional order. Communications in Nonlinear Science and Numerical Simulation, vol. 15, no. 3, pp. 616-628.

[8] Ahmed, E. & Elgazzar, A. (2007). On fractional order differential equations model for nonlocal epidemics. Physica A: Statistical Mechanics and its Applications, vol. 379, no. 2, pp. 607-614.

[9] Peng, B., et al. (2009). Differential evolution algorithm-based parameter estimation for chaotic systems. Chaos, Solitons & Fractals, vol. 39, no. 5, pp. 2110-2118.

[10] Parlitz, U. (1996). Estimating model parameters from time series by autosynchronization. Physical Review Letters, vol. 76, no. 8, pp. 1232.

[11] Wu, X., Hu, H. & Zhang, B. (2004). Parameter estimation only from the symbolic sequences generated by chaos system. Chaos, Solitons & Fractals, vol. 22, no. 2, pp. 359-366.

[12] Modares, H., Alfi, A. & Fateh, M. M. (2010). Parameter identification of chaotic dynamic systems through an improved particle swarm optimization. Expert Systems with Applications, vol. 37, no. 5, pp. 3714-3720.

[13] Jiang, Y., et al. (2016). Parameter Identification of Chaotic Systems by a Novel Dual Particle Swarm Optimization. International Journal of Bifurcation and Chaos, vol. 26, no. 2, pp. 1650024.

[14] Al-Assaf, Y., El-Khazali, R. & Ahmad, W. (2004). Identification of fractional chaotic system parameters. Chaos, Solitons & Fractals, vol. 22, no. 4, pp. 897-905.

[15] Yuan, L., Yang, Q. & Zeng, C. (2013). Chaos detection and parameter identification in fractional-order chaotic systems with delay. Nonlinear Dynamics, vol. 37, no. 2, pp. 439-448.

[16] Tang, Y., et al. (2012). Parameter identification of commensurate fractional-order chaotic system via differential evolution. Physics Letters A, vol. 376, no. 4, pp. 457-464.

[17] Mathieu, B., Le Lay, L. & Oustaloup, A.. (1996). Identification of non integer order systems in the time domain. in CESA'96 IMACS Multiconference: computational engineering in systems applications.

[18] Djouambi, A., Voda, A. & Charef, A.. (2012). Recursive prediction error identification of fractional order models. Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 6, pp. 2517-2524.

[19] Poinot, T. & Trigeassou, J.-C. (2004). Identification of fractional systems using an output-error technique. Nonlinear Dynamics, vol. 38, no. 4, pp. 133-154.

[20] Aoun, M., et al. (2007). Synthesis of fractional Laguerre basis for system approximation. Automatica, vol. 43, no. 9, pp. 1640-1648.

[21] Cois, O., et al. (2001). Fractional state variable filter for system identification by fractional model. in Control Conference (ECC), 2001 European. IEEE.

[22] Dai, Y., et al. (2016). Modulating function-based identification for fractional order systems. Neurocomputing, vol. 173, no. 1, pp. 1959-1966.

[23] Moshar Movahhed, A., et al. (2017). Fractional Modeling and Analysis of Buck Converter in CCM Mode Peration. Journal of AI and Data Mining, vol. 5, no. 2, pp. 327-335.

[24] Trigeassou, J., et al. (1999). Modeling and identification of a non integer order system. in Control Conference (ECC), 1999 European. IEEE.

[25] Holland, J. H. (1975). Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence: U Michigan Press.

[26] Jong, D. & Alan, K.. (1975). Analysis of the behavior of a class of genetic adaptive systems, Engineering. College of-Technical Reports, University of Michigan.

[27] Jothiprakash, V. & Shanthi, G. (2006). Single reservoir operating policies using genetic algorithm. Water Resources Management, vol. 20, no. 6, pp. 917-929.

[28] Tang, H.-W., et al. (2010). Parameter identification for modeling river network using a genetic algorithm. Journal of Hydrodynamics, ser. B, vol. 22, no. 2, pp. 246-253.

[29] Dai, H.-c. & Wang, L.-l.. (2006). Study and application of the inverse problem on hydraulics. journal-sichuan university engineering science edition, vol. 38, no. 1, pp. 15.

[30] Shamshirband, S., et al. (2014). A solution for multi-objective commodity vehicle routing problem by NSGA-II. in Hybrid Intelligent Systems (HIS), 2014 14th International Conference on. IEEE.

[31] Kalantari, S., et al. (2011). Routing in wireless sensor network based on soft computing technique. Scientific Research and Essays, vol. 6, no. 21, pp. 432-4441.

[32] Premalatha, K. & Natarajan, A.. (2009). Hybrid PSO and GA for global maximization. Int. J. Open Problems Compt. Math, vol. 2, no. 4, pp. 597-608.

[33] Shi, Y. & Eberhart, R.. (1998). A modified particle swarm optimizer. in Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence., The 1998 IEEE International Conference on. IEEE.

[34] Kennedy, J., et al. (2001). Swarm intelligence.: Morgan Kaufmann.

[35] Clerc, M. & Kennedy, J. (2002). The particle swarm-explosion, stability, and convergence in a multidimensional complex space. IEEE transactions on Evolutionary Computation, vol. 6, no. 1, pp. 58-73.

[36] Zheng, Y., et al. (2003). Empirical study of particle swarm optimizer with an increasing inertia weight. in Evolutionary Computation, 2003. CEC'03. The 2003 Congress on. IEEE.

[37] Zheng, Y.-L., et al. (2003). On the convergence analysis and parameter selection in particle swarm optimization. in Machine Learning and Cybernetics, 2003 International Conference on. IEEE.

[38] Larose, D.T. (2006). Data mining methods & models. 2006: John Wiley & Sons.

[39] Petras, I. (2011). Fractional-order nonlinear systems: modeling, analysis and simulation: Springer Science & Business Media.

نشریه هوش مصنوعی و داده کاوی

شناسایی سیستم‌های غیرخطی مرتبه کسری آشوبگونه با مشتق مرتبه کسری تقریبی براساس روش ترکیبی الگوریتم ژنتیک-بهینه‌سازی ازدحام ذرات

مصطفی کوثری[1] و محمد تشنه لب[2,*]

[1] آزمایشگاه سیستم های هوشمند، دپارتمان گروه مهندسی برق-کنترل، دانشگاه صنعتی خواجه نصیرالدین طوسی، تهران، ایران.

۲ گروه مهندسی برق-کنترل، دانشگاه صنعتی خواجه نصیرالدین طوسی، تهران، ایران.

ارسال ۲۰۱۶/۰۸/۳۰؛ بازنگری ۲۰۱۶/۱۲/۱۲؛ پذیرش ۲۰۱۷/۰۷/۱۹

چکیده:

اگرچه در طی سال‌های قبل بسیاری از ریاضیدانان بر روی حسابان مرتبه کسری تحقیق کرده‌اند، اما کاربرد آن در مهندسی، مخصوصاً در مدل‌سازی و کنترل، سابقه زیادی ندارد. بدلیل وجود آزادی بیشتر در انتخاب مرتبه مشتق‌گیر و انتگرال‌گیر در حسابان کسری، امکان مدل‌سازی سیستم‌های فیزیکی را دقیق‌تر فراهم می‌کند. این مقاله مرتبط است با شناسایی سیستم‌های آشوبگونه مرتبه کسری در بعد زمان، که مشتق مرسوم توسط نوع کسری آن با استفاده از مشتق غیر صحیح جایگزین می‌شود. این عملگر خودش توسط یک سیستم N-بعدی تشکیل شده از یک انتگرال گیر و یک فیلتر پیش-فاز تقریب زده می‌شود. روش ترکیبی الگوریتم ژنتیک(GA)-بهینه‌سازی ازدحام ذرات(PSO) به کار برده می‌شود تا پارامترهای سیستم غیرخطی مرتبه کسری آشوبگونه مدل شده توسط یک نمایش فضای حالت را تخمین بزند. امکان پذیر بودن این روش توسط شناسایی پارامترهای سیستم تقریب‌زده شده آشوبگونه لرنز مرتبه کسری نشان داده می‌شود. عملکرد الگوریتم بیان شده با الگوریتم ژنتیک و بهینه‌سازی ازدحام ذرات استاندارد به لحاظ دقت پارامتر و تابع هزینه مقایسه می‌شود. به منظور ارزیابی دقت شناسایی، خطای خروجی در بعد زمان به عنوان تابع برازش برای بهینه‌سازی پارامترها مطرح می‌شود. نتایج شبیه‌سازی نشان می‌دهد که روش بیان شده برای شناسایی پارامتر سیستم‌های آشوبگونه مرتبه کسری موفق‌تر از الگوریتم‌های دیگر است.

کلمات کلیدی: شناسایی پارامتر، سیستم آشوبگونه، بهینه‌سازی ازدحام ذرات، الگوریتم ژنتیک، حسابان کسری.