Journal of AI and Data Mining
Vol. 2, No. 2, 2014, 113-123.

A new methodology for deriving the efficient frontier of stocks portfolios: An advanced risk-return model

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Received 23 April 2013; accepted 07 December 2013
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Abstract
In this paper, after reviewing the concept of Efficient Frontier (EF), an important inadequacy of the Variance based models for deriving EFs and the high necessity for applying another risk measure is exemplified. To meet the challenge, the traditional risk measure of Variance is replaced with Lower Partial Moment (LPM) of the first order. Because of the particular shape of the new risk measure, one part of the paper is devoted to a methodology for deriving EF on the basis of the new model. Then the model superiority over the old one is shown and finally shape of the new EFs under different situations is investigated. At last, it is concluded that application of LPM of the first order in financial models in the phase of deriving EF is completely wise and justifiable.

Keywords: *Efficient Frontier, Portfolio Optimization, Markowitz Model, Lower Partial Moment Model, Genetic Algorithm.*

1. Introduction
The portfolio optimization problems have been one of the important research fields in modern financial knowledge. Investors including large institutions such as mutual funds and pension funds use portfolio management systems to support their asset allocations. In this regard, deriving EF on the basis of historical information is an essential initial step to remove inefficient portfolios otherwise the complexity of decision making increases considerably. A portfolio is efficient if there is no other portfolio with the same or higher expected return that has lower risk, the collection of portfolios with this property is called efficient set or efficient frontier. On the important position of EF in field of portfolio selection, it is good to refer to Ballestero and Romero [1] and Jasemi et al. [2] that recommend maximizing investors' expected utility on EF to come to the best choice for the investment.
Our study is categorized in the third direction with a risk measure of Lowe Partial Moment of the first order. The mean–variance objective function may not be the best choice available to investors in terms of an appropriate risk measure. Furthermore, other risk measures may be more appropriate. In this category of literature, when it comes to our selected risk measures that are sensibly coherent by Jasemi et al. [3] the literature has been essentially a vacant area while a good attention has been directed to the other risk measures. According to Bertsimas et al. [4], two main problems of LPM are computational and the fact that standard portfolio theory results have been broaden to the LPM risks only for some special values of τ or for some special families of distributions.
Achieving long-term and sustained economic growth requires optimized preparation and allocation of resources at economic level and this is not possible without a help from financial market especially an efficient and extensive capital market. In a healthy economy, the presence of an efficient financial system plays an essential role in appropriate distribution of capital

and financial resources. Risk measures always play a significant role in financial model especially in portfolio optimization and rarely can a model be found without risk measures. When the importance of efficient frontier is known in the financial literature, critical role of risk measures becomes apparent clearly.
In relation to capital markets, extensive research has been done with the aim of maximization of investor's satisfaction in portfolio selection models. The development of these models is a difficult task because social, political and economic variables influencing capital markets are not predictable. However, the more realistic the risk measure, the more free the selection of return measure. In the present paper, first order LMP is put in a risky place.
Then, using models with first order LMP risk measures, we concentrate on calculation and approximation challenges of drawing efficient frontier. Indeed, a mechanism is provided for drawing efficient frontier.
But with respect to increasing complexity of calculations and presence of many influential factors in calculation of results, using traditional search methods and individual examination of each potential solution would not be appropriate; thus in recent decades there has been an increased tendency towards methods based on natural life such as evolutional algorithms of neural networks, ant algorithms and genetic algorithms.
Following rapid development of various science branches in 20th century, non-linear functions were developed in different engineering processes which need to numerical solution for them led to evolution of various structures for numerical solutions. Genetic algorithm as one of these structures was also created around three decades ago inspiring from natural structures [5].
The most significant qualities of each numerical algorithm are: 1) generalizability, 2) convergence speed and 3) solution accuracy which in genetic algorithm the first one is in a good condition and this algorithm is approximately generalizable to every engineering structure. However, parts 2 and 3 are usually in the opposite directions and the improvement in one of them leads to deterioration of the other.
In many cases, there is a simultaneous need to high accuracy and convergence speed, because of high computational volume and relative weakness of methods or impossibility of accurate determination of some parameters of the algorithm, solution time number of iterations before achieving solution greatly increases. With respect to the fact that new applications need to both qualities, various methods were proposed to simultaneous improvement of 2 and 3 among them [6,7,8,9] can be mentioned.
With respect to the above, in section 1 of the present research, some important risk measures are addressed. Then in section 2, LPM is studied in a complete manner and in section 3, a method for obtaining efficient frontier of optimum portfolio based on LPM is provided and it is shown that by replacing model variance by LPM, the model turns into an NP-hard one and as the result of this, genetic algorithm is used to solve it. In section 4, data are analyzed and in final section research is concluded.

2. Risk measures

Risk measures have always played a significant role in financial models especially in portfolio optimization family and a model without a risk measure can rarely be found. In the field of portfolio theory, variance, semi-variance, adverse outcome probability, value at risk (VaR) and conditional value at risk (CVaR) and LPM are among the most well-known risk measures [2].

2.1. Variance

Variance is the most acceptable definition for risk. According to this definition, if r is asset return and μ is expected value, risk of asset investment is:

$$V[r] = E\left[(r-\mu)^2\right] \tag{1}$$

2.2. Semi-variance

This measure evaluates variability of below-average returns. Mathematical description for semi-variance is as follows:

$$SV[r] = E\left[\left[(r-\mu)^-\right]^2\right] \tag{2}$$

Where

$$(r-\mu)^- = \begin{cases} r-\mu, & if \quad r \le \mu \\ 0 \quad , & if \quad r > \mu. \end{cases} \tag{3}$$

2.3. Adverse outcome probability

This measure defines risk as the probability that asset value becomes lower than a certain level. If b and r_0 respectively indicate a fixed value and distance:

$$\Pr\left\{(b-r) \ge r_0\right\} \tag{4}$$

According to downside risk measure, the above probability and (b-r) are respectively known as risk and loss of the investment.

2.4. Value at Risk

Risk at value measure is very similar to adverse outcome probability in such a way that Huang [10] views it as another description for adverse outcome probability. If β is a predefined value, VaR_β is the portfolio with the least value of α in such a way that with a probability of 1- β, investment loss would be lower than α [2].

2.5. Conditional value at risk

Some unfavorable characteristics of value at risk including lack of Sub-additivity and convergence led to development of conditional value at risk measure by Rockafellar et al. [2].

3. LPM

The portfolio optimization problems have been one of the important research fields in modern financial knowledge. Investors including large institutions such as mutual funds and pension funds use portfolio management systems to support their asset allocations. In this regard, deriving efficient frontier (EF) on the basis of historical information is an essential initial step to remove inefficient portfolios otherwise the complexity of decision making increases considerably. On the important position of EF in the field of portfolio selection, it is good to refer to Ballestero et al. [1], [2].

Symmetric risk measures such as Removed variance are generally downside measures such as LMP [11]. One class of downside risk measures which are consistent with definition of increasing risk for optional probability distribution is LPM. Attractiveness of these risk criteria are to some extent due to their consistency with the way the risk being perceived by individuals [12] and thus LMP approach is of significant importance for financial decision making.

This class of risk measures is of significant efficiency from both theoretical and practical perspective. Bawa [13] showed that for each scalar amount R_{nec} and for each return distribution belonging to a specific class of distributions, LPM average model creates portfolios which are superior to other portfolios according to probable dominance concept.

Bawa [13] introduced a general definition of downside risks in form of lower partial moment (LPM) and Fishburn [14] developed the (α, R_{nec}) model. This measure of order α around R_{nec} is defined in (5).

$$LPM_\alpha\left(R_{nec};R\right) \equiv \int_{-100}^{R_{nec}} \left(R_{nec}-R\right)^\alpha dF(R) = E\left\{\left(\max\left[0, R_{nec}-R\right]\right)^\alpha\right\} \qquad (5)$$

Where $F(R)$ is cumulative distribution function of the investment return R, R_{nec} is the target parameter. By changing the parameters of α R_{nec} different risk measures can be developed. In this study, LPM1 (R_{nec}, R) that according to Fishburn [11] concerns a risk-neutral investor and has been discussed in some aspects by Spreitzer et al. [5] is used as is shown by (6).

$$\int_{-100}^{R_{nec}} (R_{nec} - R) f\,(R) dR. \qquad (6)$$

On the field of deriving EF by a risk measure other than the famous variance, except for the family of LPM, the literature is full. The models that are based on the semi-variance are such as Homaifar and Graddy [10], Markowitz [15], Rom and Ferguson [16], Chow and Denning [9], Grootveld and Hallerbach [17] and Enrique [18] that proposed a semi-variance based EF model. Konno et al. [19] showed large scale mean semi variance models are solvable by mathematical programming or Huang [20] developed a fuzzy Mean semi variance model. About the mean absolute deviation, Konno [21]; and Konno and Yamazaki [22] first proposed a mean absolute deviation portfolio optimization model while the model can be solved by linear methods. On the basis of this model, Speranza [5] introduced a model with a weighted risk function considering minimum transaction lots and maximum number of securities. Mansini and Speranza [23] regarded transaction costs with and without minimum transaction lots based on Konno model. Konno [24] and Konno and Koshizukawell [7] discussed the computational advantages of it over the Markowitz model. Another alternative definition of risk is the probability of an adverse outcome that parallel to the publication of Markowitz model developed by Roy [25] Mao [26] and Williams [27] minimizing the probability of an adverse outcome. Ortobellil and Rachev [8] studied the stable Paretian approach and the safety-first analysis in portfolio selection theory in coherence with the empirical evidence and the stochastic dominance theory and Rambaud et al. [28] focus on new considerations on the classical models and many other valuable works. However, there is scarcity in the literature focusing on LPM. According to Bertsimas et al. [4], two main problems of LPM are computational difficulties and the fact that standard portfolio theory results have been broaden to the LPM risks only for some special values of R_{nec} or for some special families of distributions.

4. The methodology

4.1. The classical EF model

Portfolio is to deal with the problem of how to allocate wealth among several assets. The classical EF model, which was firstly developed by Markowitz [29] is as follows:

$$Min \quad Risk\left(P\left(x_1,...,x_n\right)\right)$$

s.t:

$$\sum_{i=1}^{n} x_i = 1$$

$$\sum_{i=1}^{n} x_i \bar{r}_i = R_d$$

$$\sum_{i=1}^{n} y_i = a$$

$$l_i y_i \le x_i \le u_i y_i \qquad i = 1,...,n$$

$$y_i = \begin{cases} 1 & x_i > 0 \\ 0 & x_i = 0 \end{cases}$$

$$x_i \ge 0 \qquad i = 1,2,...,n$$

Where, Risk: Risk function.

x_i: Share of stock i in the portfolio.

$P(x_1,...,x_n)$: The portfolio whose shares of stocks are $x_1,...,x_n$.

$\bar{r}_i$: Indicator of stock i past performance from the perspective of return.

a: Desired number of stocks in the portfolio.

l_i: Lower limit for share of stock i in the portfolio.

u_i: Upper limit for share of stock i in the portfolio.

3.2. The LPM of the first order of a portfolio to calculate LPM_1 (R_{nec}, R) of $P(x_1,...,x_n)$, the approach by (7) is applied.

$$\int_{-100}^{R_{nec}} \left(R_{nec} - \sum_{i=1}^{n} x_i r_i\right) f_{RP\left(x_1,...,x_n\right)}\left(\sum_{i=1}^{n} x_i r_i\right) d\left(\sum_{i=1}^{n} x_i r_i\right) \tag{7}$$

Where $RP(x_1,...,x_n)$ is return of the portfolio with shares of $x_1,...,x_n$.

The first step to approximate $LPM_1(R_{nec}, RP(x_1,..,x_n))$ on the basis of (7) is estimation of $f_{RP}(x_1,..,x_n)$ that in this study will be done by drawing the associated histograms. If it is assumed that the time horizon is of length T, return of a portfolio with shar $x_1,..., x_n$ on the t^th time unit is calculated by (8).

$$RP_t\left(x_1,...,x_n\right) = x_1.ar_{1t} + x_2.ar_{2t} + ... + x_n.ar_{nt} = \sum_{i=1}^{n} x_i.ar_{it} \quad t = 1,2,...,T \tag{8}$$

Where ar_{it} is the return of asset i on the t^{th} time unit. After calculating, the portfolio returns for all the T time units according to (8), the necessary data to draw a histogram are available while in the intended histogram, the intervals are too short to encompass more than one distinct data; i.e. two different data surely fall in two separate intervals. Naturally, here the data itself represents the interval. Bowker et al. [30] has referred to the strategy by saying that if instead of frequency of each interval, the number of observations that relates to a distinct data is cited, better results can be achieved. Figure1 shows a typical histogram of this kind while r_i denotes the i^{th} smallest return of the asset, f_i determines frequency of r_i and N is the number of different returns of the asset.

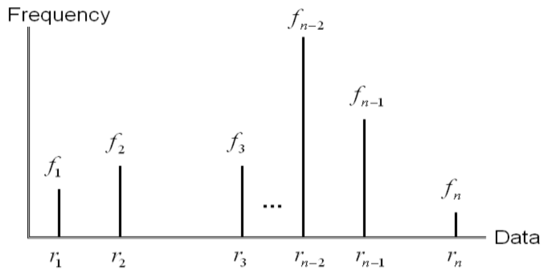

Figure 1. A typical histogram that is drawn by the first strategy [2].

Since the histograms based on this strategy are discrete, calculation of the LPM in its continuous form of (6) is not possible and the equation should be converted to its equivalent discrete one as (9).

$$LPM_1\left(R_{nec}, R\right) \approx \sum_{-100}^{R_{nec}} \left(R_{nec} - R\right) p\left(R\right). \tag{9}$$

Where $p(R)$ is the associated probability function and the steps of the sigma are determined by the $r_1,..., r_n$ Now if $r_k < \tau < r_{k+1}$, $LPM(R_{nec}, R)$ is calculated by (10).

$$\frac{\sum_{i=1}^{k}\left(R_{nec} - r_{pi}\left(x_1,...,x_n\right)\right) f_i}{\sum_{i=1}^{N} f_i} \tag{10}$$

Based on what has been discussed, the final optimization model, however in its simplest form without the optional constraints, to get the intended EF is as follows.

$$Min\ \ LPM_1\left(R_{nec},RP\left(x_1,...,x_n\right)\right)=\frac{\sum_{i=1}^{k}\left(R_{nec}-r_{pi}\left(x_1,...,x_n\right)\right)f_i}{\sum_{i=1}^{N}f_i}$$

$$\sum_{i=1}^{n}x_i=1$$

$$\sum_{i=1}^{n}x_i\bar{r}_i=R_d$$

$$x_i\geq 0\quad i=1,2,...,n$$

Where r_{pi} $(x_1,\ldots,x_n)$, is ith smallest return for portfolio with investment percentages of $x_1,\ldots x_n$

In above model, parameters k and r_{pi} are non-linear functions of problem decision variables $(x_1,x_2,\ldots,x_n)$. Solving many non-linear problems is a time-consuming and complex task and with increase in problem size, the solution time increases in an exponential manner and makes it harder. Thus our problem is an NP-hard one. Today, to solve many of such problems heuristic algorithms especially genetic one are used and present study also exploits genetic algorithm.

5. Solution algorithm

The model obtained in the formula is of a high computational complexity. Solving these types of problems is difficult using accurate methods. Genetic algorithm is a powerful tool to solve such type of models [31].

5.1. Genetic algorithm

Based on the Darwin principle ‘‘the fittest survive” in nature, genetic algorithm (GA) was first initiated by Holland [32] sand has rapidly become the best-known evolutionary techniques Goldberg [33]. Since the pioneering method by Holland, numerous related GA-based portfolio selection approaches have been published. Arnone, Loraschi, and Tettamanzi [34] presented a GA for the unconstrained portfolio optimization problem with the risk associated with the portfolio being measured by downside risk. Lin and Liu [35] proposed that GA for portfolio selection problems with minimum transaction lots. Chang et al. [36] try to solve three models separately with risk measures of semi-variance, mean absolute deviation and variance with skewness by genetic algorithm. Soleimani [3] consider Markowitz model with three constraints of Minimum transaction lots, cardinality constraints and market capitalization and solved it by genetic algorithm. Recently, GA has attracted much attention in portfolio optimization problems. In GA, an initial population containing constant number of chromosomes is generated randomly. With regard to portfolio optimization problems, each chromosome represents the weight of individual stock of portfolio and is optimized to reach a possible solution. An evaluation function is formed to evaluate the fitness for each chromosome, which defines how good a solution the chromosome represents. By using crossover, mutation values and natural selection, the population will converge to one containing only chromosomes with good fitness. Where the larger the fitness value is, the better objective function value the solution has. The basic steps in GA are shown as follows:

Step 1: Initialize a randomly generated population.

Step 2: Evaluate fitness of individual in the population.

Step 3: Apply elitist selection: carry on the best individuals to the next generation from reproduction, crossover, and mutation.

Step 4: Replace the current population by the new population.

Step 5: If the termination condition is satisfied then stop, else go to Step 2.

Through this reproduction once, the children of two chromosomes are generated. The reproduction process is operated until all chromosomes of a new population have been generated thoroughly. Through specified maximum generations, the best solution ever found is the answer.

Genetic algorithm is a search technique in computing science which aims to find an approximate way for optimization. Genetic algorithm is a specific type of evolutional algorithms which uses such techniques as inheritance and mutation [37]. It was initially proposed by john Holland [38] in 1960 but its usual form was provided by Goldberg [39]. Genetic algorithm is an innovative search method which follows evolutional trend of nature based on Darwin theory.

5.2. Fitness estimation and primary population

In this phase, a primary population (a set of chromosomes) is generated in a random manner. Some of them do not meet equation constraints. Thus, the production of chromosomes is controlled using death penalty method in order to achieve reasonable chromosomes.

5.3. GA operators

GA operators such as crossover and mutation ones contribute to generation of the next population.

In crossover process, a pair of mature chromosomes should generate two children. This is done randomly by a pair of chromosomes from the same generation with probability of $P_{c.}$ In present research, two point crossover method is used. In mutation, a chromosome is randomly selected from the population and position of one gene of it is replaced by random selection of a number within parameter range.

5.4. Selection of chromosome and stop condition

Reasonable chromosomes have to compete on selection in the next phase. Selection operator chooses chromosomes from existing population for the next phase based on their fitness value. Several selection method such as roulette wheel, tournament selection, rank selection, elitism selection have been mentioned in investigations by Michalewicz [40].

Thus N_{pop} chromosomes are selected among parents and children with the most fitness. In GA, stop condition is the last step. In present research, a certain number of generations are used given the parameter setting in test design.

In present study, running genetic algorithm was done using MATLAB software.

5.5. Methodology of comparison of two models

The main goal of present section is to show the major drawback of mean-variance models for extraction of efficient frontier.

Consider a situation in which there are only two stocks with returns of r_1 and r_2 in a way that r1>r2. If an investor wants to form the best portfolio from r1 and r2, which combination of them should be selected?!

In order to examine performance of variance-based efficiency frontier models in such situations, a financial period of 301 days with 300 positive returns of r1 and r2 is considered where r2= r1\2.

$$Min\ \operatorname{var}\left(x_1 r_1 + x_2 r_2\right) = x_1^2 \operatorname{var}\left(r_1\right) + x_2^2 \operatorname{var}\left(r_2\right) + 2x_1 x_2 \operatorname{cov}\left(r_1, r_2\right)$$
$$= 0.0107x_1^2 + 0.0027x_2^2 + 0.0107x_1 x_2$$

St:

$$x_1 + x_2 = 1$$
$$0.6053x_1 + 0.3027x_2 = R_d$$
$$x_1, x_2 \geq 0$$

Table1. Portfolio forming the efficient frontier.

Point	X_1	X_2	variance	R_d
1	0.9957	0.0053	0.0106	0.6043
2	0.9434	0.0573	0.0098	0.5884
3	0.8905	0.1104	0.009	0.5725
4	0.8381	0.1628	0.0083	0.5566
5	0.7852	0.2158	0.0076	0.5406
6	0.7329	0.2677	0.007	0.5247
7	0.6799	0.321	0.0064	0.5087
8	0.6273	0.3736	0.0058	0.4928
9	0.5747	0.4263	0.0053	0.4769
10	0.5221	0.4789	0.0049	0.461
11	0.4694	0.5316	0.0044	0.445
12	0.4168	0.5842	0.0041	0.4291
13	0.3642	0.6367	0.0037	0.4132
14	0.3129	0.6867	0.0035	0.3972
15	0.2609	0.7381	0.0032	0.3813
16	0.2082	0.7908	0.003	0.3654
17	0.1556	0.8434	0.0029	0.3495
18	0.103	0.896	0.0028	0.3335
19	0.051	0.948	0.0027	0.3178
20	0.0011	0.9979	0.0027	0.3027

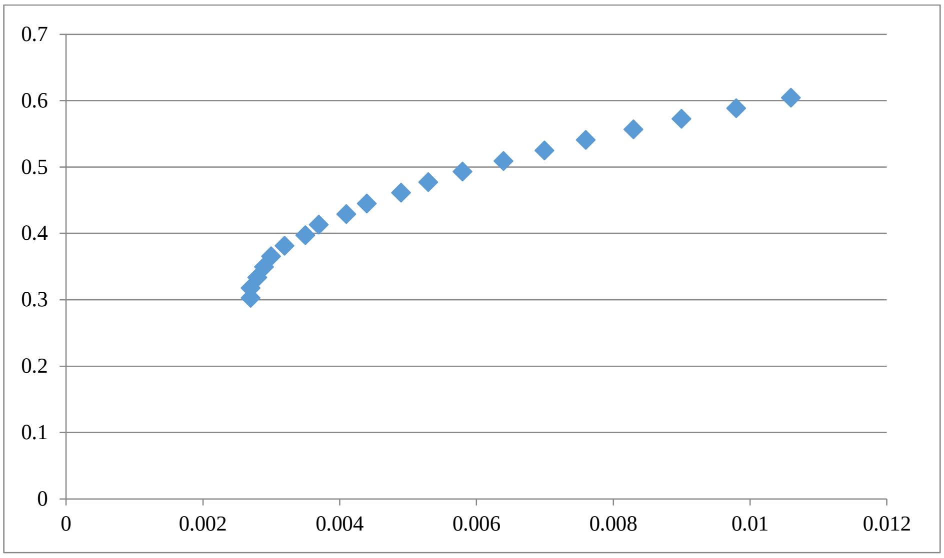

Figure 2. Efficient frontier based on data of Table 1.

Results from table 1 are also plotted in figure 2. As can be seen, the model proposes more than one point while the most surprising fact is that even an investment of 100 percent in r_2, which is never justifiable in practice, has been proposed. It should be mentioned that columns x_1 and x_2 of table1 indicate percentage investment in r_1 and r_2, respectively.

5.6. New efficient frontier model (LPM)

In order to see performance of new efficient frontier model with respect to case presented in section 5.4, the model was initiated with R_{nec} = 20%.

Table2. Portfolio forming efficient frontier with LPM and Rnec= 20%.

Point	X_1	X_2	LPM	R_d
1	0.0032	0.9969	0.0006	0.3071
2	0.0566	0.9427	0.0004	0.3228
3	0.1088	0.891	0.0003	0.3386
4	0.1614	0.8384	0.0002	0.3543
5	0.2149	0.7842	0.0001	0.3701
6	0.2667	0.733	0	0.3858
7	0.3201	0.6789	0	0.4016
8	0.3684	0.6316	0	0.4163
9	0.4211	0.5789	0	0.4321
10	0.4737	0.5263	0	0.4478
11	0.5263	0.4737	0	0.4636
12	0.5789	0.4211	0	0.4793
13	0.6316	0.3684	0	0.4951
14	0.6842	0.3158	0	0.5108
15	0.7368	0.2632	0	0.5266
16	0.7895	0.2105	0	0.5423
17	0.8421	0.1579	0	0.5581
18	0.8947	0.1053	0	0.5738
19	0.9474	0.0526	0	0.5896
20	1	0	0	0.6053

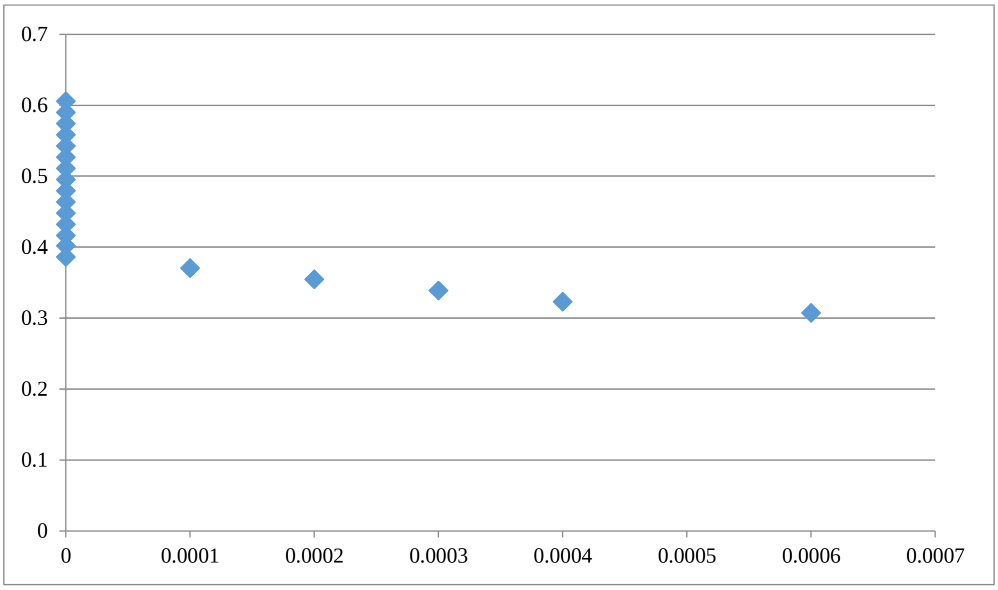

Figure 3. Efficient frontier of LPM model with Rnec= 20%.

According to definition of efficient frontier and figure3, for each R_{nec}, only the last point in left side which suggests an investment of 100% can be accepted as efficient frontier.
Thus, mean LPM model has produced the most acceptable results.

5.7. Analysis of model data

After examining the model and after LPM model turned into a NP-hard model, we used genetic algorithm to obtain efficient frontier based on data from New York Exchange and implementation of it was done using MATLAB software.

In order to run the algorithm, we used monthly prices of 20 firms listed on New York Stock Exchange. In this way, price returns were calculated and using them, previous performance index for portfolio was calculated.
Thus, in order to obtain efficient frontier and observe the trend and results of them, each time some pairs were selected from 20 stocks with the aim of deriving efficient frontier and this trend was repeated up to 16 stocks. For example, at first, 8 stocks from total 20 ones were selected and this process continued up to 16 stocks.

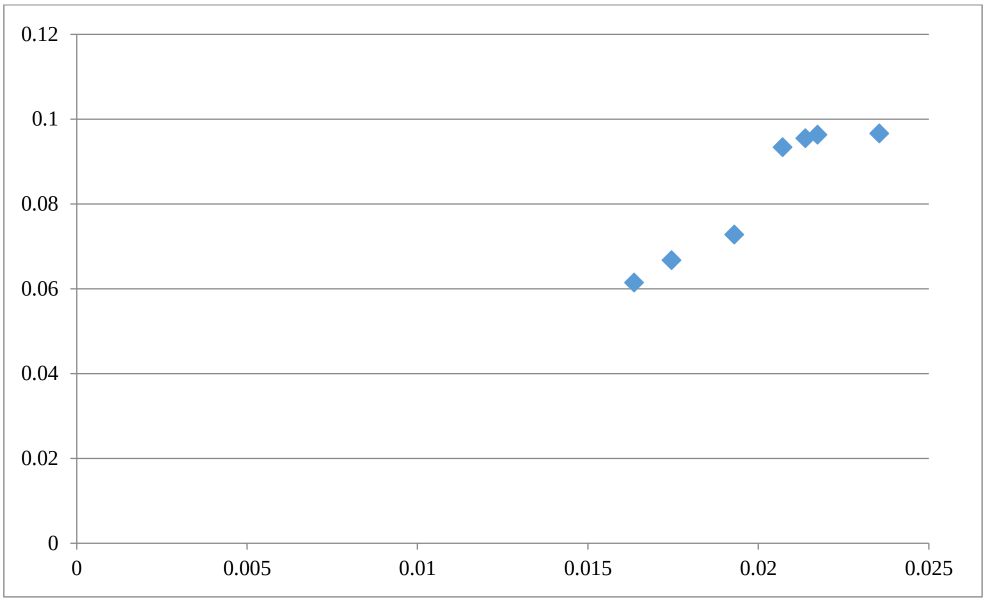

Figure 4. Efficient frontier with constraint of selecting 8 stocks from 20 ones for LPM model with Rnec= 0.

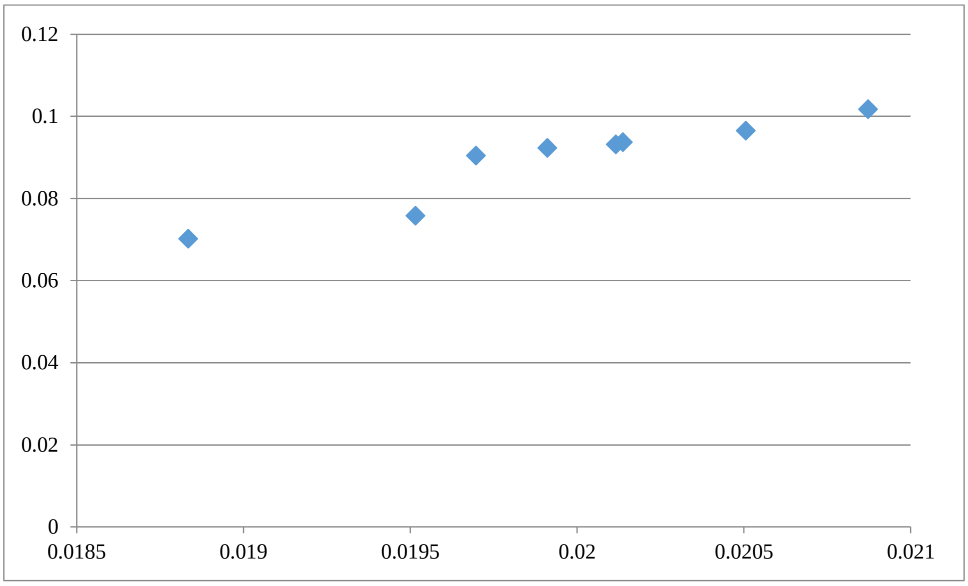

Figure 5. Efficient frontier with constraint of selecting 10 stocks from 20 ones for LPM model with Rnec= 0.

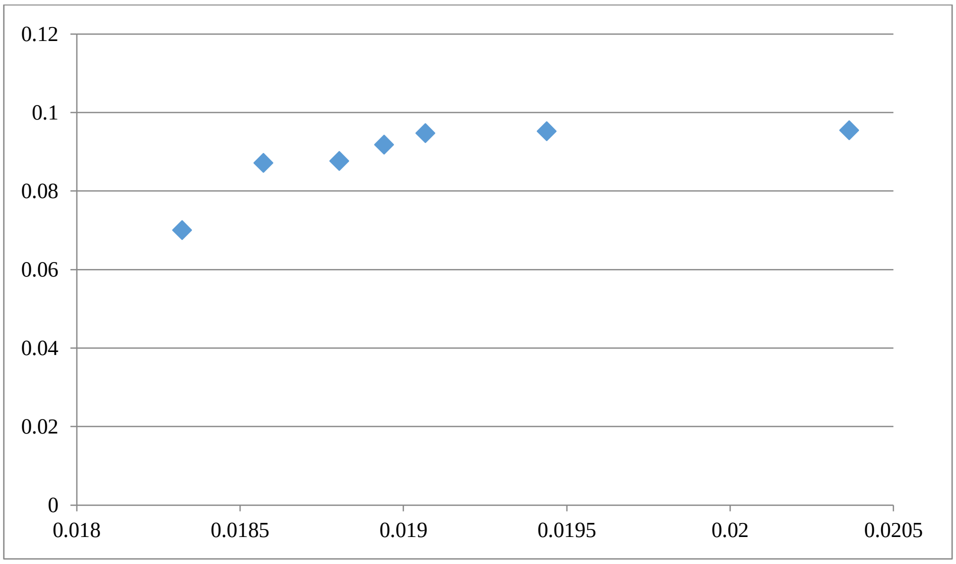

Figure 6. Efficient frontier with constraint of selecting 12 stocks from 20 ones for LPM model with Rnec= 0.

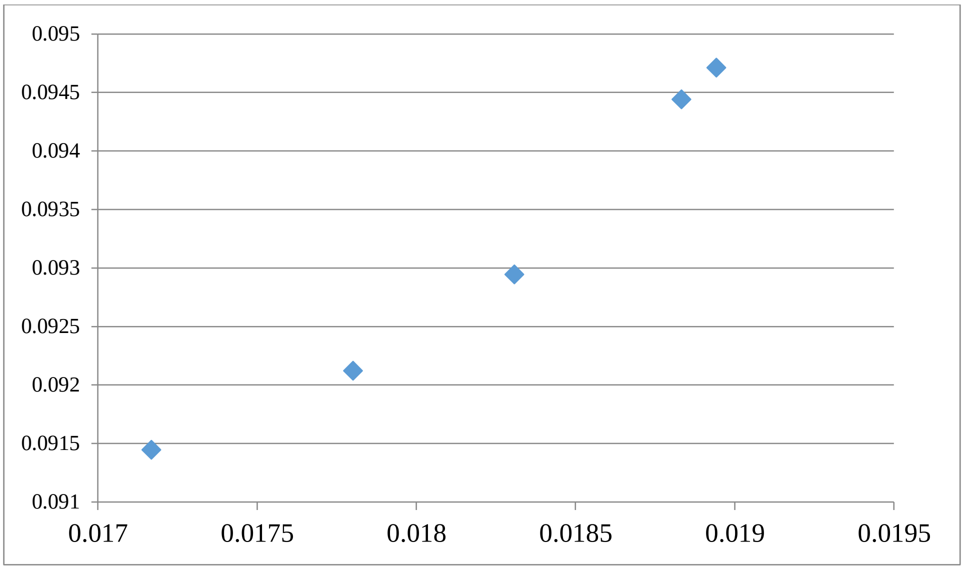

Figure 7. Efficient frontier with constraint of selecting 14 stocks from 20 ones for LPM model with Rnec= 0.

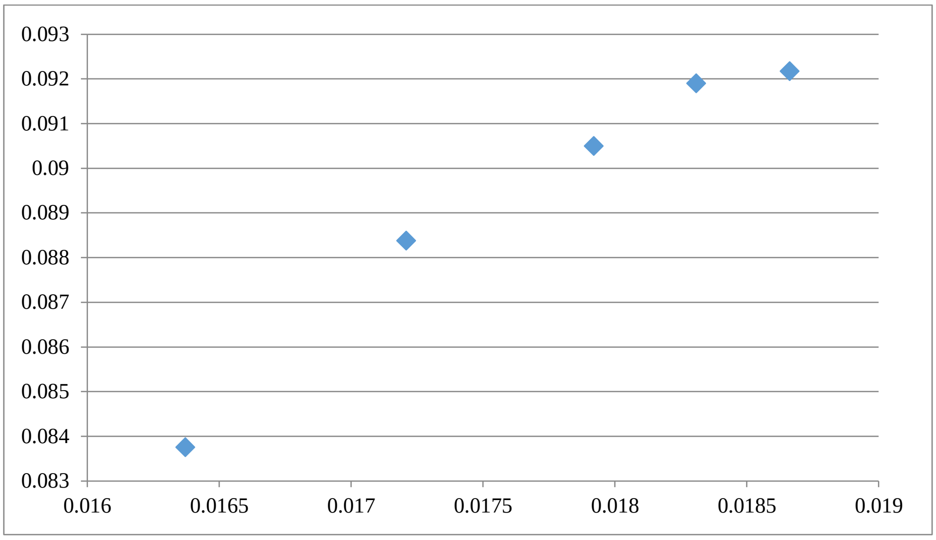

Figure 8. Efficient frontier with constraint of selecting 16 stocks from 20 ones for LPM model with Rnec= 0.

6. Conclusion

The concept of EF was the main focus of this paper, and the difference between this study and the others of the field can be summarized in the two following items.

- Considering the risk measure of LPM of the first order for deriving EF.
- Presenting a practical approach to derive EF on the basis of the LPM while the approach is not restricted by factors like stochastic characteristics of the stocks returns or number of stocks that compose the portfolio. The attained results extracted from previous research suggest that applying the lower partial moment to financial model in the stage of obtaining efficient frontier given clear and complete market constraints is more rational and justified. In present study, two models of Markowitz variance and LPM were implemented using genetic algorithm and a comparison was done between them. Then, efficient frontier was gained for 20 firms listed on New York Stock Exchange. On the other hand, graphs presented in section 4 indicate that efficient frontier of portfolio is concaved and also our efficient frontier obtains acceptable order with an increase in stock number. Surely, this trend is not always the case and it is possible that with an increase in number of stocks, efficient frontier becomes afflicted with disorder. Finally, the simulation results prove that the LMP model have high efficiency compared with other models.

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نشریه هوش مصنوعی و داده کاوی

دانشگاه شاهرود

ارائه‌ی یک متدولوژی جدید برای استخراج مرز کارای پورتفوی سهام بوسیله‌ی یک مدل ریسک-بازده توسعه‌یافته

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ارسال ۲۰۱۳/۰۴/۲۳؛ پذیرش ۲۰۱۳/۱۲/۰۷

چکیده:

در این مقاله بعد از مروری بر مفهوم مرز کارا، یک نقص مهم مدل‌های مبتنی بر واریانس برای استخراج مرزهای کارا و ضرورت بالای کاربرد یک معیار ریسک دیگر با مثال نشان داده شده است. برای چالش مزبور، معیار سنتی واریانس با معیار گشتاور جزئی پایینی مرتبه اول جایگزین شده است. به خاطر شکل خاص معیار جدید، بخشی از مقاله به توسعه یک روش برای استخراج مرز کارا بر اساس مدل جدید اختصاص داده شده است. سپس برتری و امتیاز مدل جدید به همتای سنتیش نشان داده می‌شود و در نهایت شکل مرز کارای جدید تحت شرایط مختلف مورد تحقیق قرار گرفته است. در پایان نتیجه‌گیری می‌شود که کاربرد گشتاور جزئی پایینی مرتبه اول در مدل‌های مالی در مرحله استخراج مرز کارا کاملا منطقی و قابل توجیه است.

کلمات کلیدی: مرز کارا، بهینه‌سازی پورتفوی، مدل مارکوویتز، مدل گشتاور جزئی پایینی، الگوریتم ژنتیک.