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A Combinatorial Algorithm for Fuzzy Parameter Estimation with Application to Uncertain Measurements

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Abstract

In this paper, we present a new method for the regression model prediction in an uncertain environment. In the practical engineering problems, to develop the regression or the artificial neural network model for making predictions, an average set of the repeated observed values is introduced to the model as an input variable. Therefore, the estimated response of the process is also the average of a set of output values, where the variation around the mean is not determinate. However, to provide the unbiased and precise estimations, the predictions are required to be correct, on average, and the spread of date should be specified. In order to address this issue, we propose a method based on the fuzzy inference system, and the genetic and linear programming algorithms. We consider the crisp inputs and the symmetrical triangular fuzzy output. The proposed algorithm is applied to fit the fuzzy regression model. In addition, we apply a simulation example and a practical example in the field of machining process in order to assess the performance of the proposed method in dealing with the practical problems in which the output variables have the natures of uncertainty and impression. Finally, we compare the performance of the proposed method with other available methods. Based on the provided examples, the proposed method is verified for prediction. The results obtained show that the proposed method reduces the error values to a minimum level, and it is more accurate than the linear programming method and the fuzzy weights with linear programming method.

Keywords: *Fuzzy Regression; Linear Programming; Machining Process; Adaptive Neuro-fuzzy Inference System; Genetic Algorithm.*

1. Introduction

In many practical engineering problems, the measured output parameters are not unique. Thus in order to facilitate the modeling of data, it is common to use the average of the measured numbers. Therefore, the predicted outputs are calculated on the basis of the average values without considering the dispersion of the dataset. Furthermore, the average values are very sensitive to outliers.

The fuzzy logic and fuzzy systems are one of the ways available to address uncertainty in engineering applications. The variations in the variable around the average values could be represented using fuzzy logic.

There are many inexact optimization methods, the most important of which are stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP), and interval linear programming (ILP). They have been developed to tackle the uncertainties. FMP has a lower data requirement but reflects a more flexible information in practical applications because the related membership functions are more easily defined [1].

The linear model of fuzzy regression analysis, established by Tanaka *et al*., has enabled the fuzzy system to give the fuzzy output [2]. In the Tanaka's research work, a fuzzy functional relationship has been given between the explanatory variables and response variables in the fuzzy regression model. In the fuzzy linear programming problems, known as the FFLP problems, all the parameters as well as the variables are represented by fuzzy numbers [3].

Lotfi et al. have discussed an FFLP problem in which all parameters and variables are triangular fuzzy numbers. They used the concept of the symmetric triangular fuzzy number and introduced an approach to defuzzify a general fuzzy quantity [4]. Kumar *et al*. have pointed out the shortcomings of the existing methods [4], and to overcome these issues, they proposed a method with equality constraints to find the fuzzy optimal solution to the FFLP problems. Goudarzi *et al*. have discussed a fully fuzzy mixed integer linear programming problem and presented a solving method [5].

Several fuzzy regression techniques such as linear programming (LP) and quadratic programming (QP) have been proposed based on the fuzzy least squares (FLS) and mathematical programming methods that minimize the total spread of the output. The FLS and LP methods have been proposed by Diamond [6] and Tanaka *et al*. (see [6-8]), respectively. For the fuzzy linear regression problem, several variants of the FLS (see [9-11]) and LP methods (see [12, 13]) have been applied. In the fuzzy literature, several extensions of these methods have been proposed.

In order to obtain the fuzzy output, Danesh *et al*. [5, 8] have used the LP and FLS methods to optimize the consequent parameters in the hybrid algorithm of the adaptive neuro-fuzzy inference system (ANFIS). A large number of research works have been carried out on the application of ANFIS modelling to uncertainty environments [14-16].

However, there have been few attempts on the application of ANFIS with the fuzzy output. In this work, we propose a new algorithm to reduce the error of the fuzzy regression model. In this algorithm, we use the fuzzy inference system and the genetic algorithm to optimize the premise parameters. Also in the proposed algorithm, LP is used for the consequence of parameter prediction. In the ANFIS method, the output is crisp. Hence using the proposed algorithm, we can employ the adaptive neuro-fuzzy inference system method for both the crisp inputs and the symmetrical triangular fuzzy output. This algorithm is compared with the method proposed by Danesh [9], which is based on the adaptive fuzzy inference system and the fuzzy weights with linear programming (FWLP). Based on the simulation and practical examples, we demonstrate that the proposed method has a lower error than the LP and FWLP methods, and it is further verified by the predictions. This paper includes five sections. After the "Introduction", the concepts and formulations of different models are explained in Section 2. The methodology of the proposed method is presented in Section 3 to obtain the premise and the consequent parameters in the fuzzy regression. In Section 4, two examples are used to illustrate the proposed procedure and its ability in providing more accurate predictions of uncertain outputs by means of this method. Of these two examples, one is a simulated example and the other is a case study in the field of turning process.

2. Basic concepts and methods

In this section, we briefly review the basic concepts of fuzzy regression, adaptive neuro-fuzzy inference system and genetic algorithm, respectively.

2.1 Fuzzy Regression

The function $f(X)$ is a mapping from x to Y, where $x_j = (x_{j0}, x_{j1}, ..., x_{jp})(j = 1, 2, ..., n)$ is a p-dimentional vector of the crisp-independent variable and the domain is assumed to be $D \subseteq R^p$.

Consider the following fuzzy regression model:

$$Y = f(X)\{+\}\varepsilon = (l(x), a(x), r(x))_{LR}\{+\}\varepsilon. \quad (1)$$

ε represents the regression error with conditional mean zero and variance $\sigma^2(x)$ given x. In this paper, the response variable Y has a symmetric triangular fuzzy structure; Y_j can be written as $Y_j = (a_j, \beta_j)$ where a_j and β_j are the center and the spread of a symmetric triangular fuzzy number, respectively, and $\beta_j = r_j - a_j = a_j - l_j$.

2.2. Linear Programming (LP) in Fuzzy Regression

In this work, we consider the following fuzzy regression model:

$$Y_j = p_0 + p_1 x_1 + p_2 x_2 + ... + p_p x_{jp} = pX_j, (j = 1, 2, ..., n) \quad (2)$$

where, n is the number of data points, $x_j = 1, x_{j1}, x_{j2}, ..., x_{jp}$ is a p-dimensional input vector of the independent variables at the j^{th} observation, $P = (p_0, p_1, .., p_p)$ is a vector of unknown fuzzy parameters, and Y_j is the j^{th} observed value of the dependent variables. P can be denoted in the vector form as $P = \{b, \alpha\}$, where

$b = (b_0, b_1, .., b_p)$ $\alpha = (\alpha_0, \alpha_1, .., \alpha_p)$, b_i is the center value, and α_i is the spread value of,. $p_i, i = 0, ..., p$ Also $Y_j = (\alpha_j, \beta_j)$ is the symmetric triangular fuzzy number, where a_j and β_j are the center and the spread of this number, respectively. Also the fuzzy regression parameters can be obtained by solving the following LP model [8]:

$$Min\, L = \sum_{j=1}^{n} \sum_{i=0}^{p} \alpha_i \left| x_{ji} \right| \tag{8}$$

Thus the following two constraints must be established:

$$\begin{aligned} &\sum_{i=0}^{p} b_i \left| x_{ji} \right| - (1-h) \sum_{i=0}^{p} \alpha_i \left| x_{ji} \right| \le \alpha_j - (1-h)\beta_j \\ &\sum_{i=0}^{p} b_i \left| x_{ji} \right| - (1-h) \sum_{i=0}^{p} \alpha_i \left| x_{ji} \right| \ge \alpha_j + (1-h)\beta_j \end{aligned} \tag{9}$$

In this model, the constrains assure that the support of the estimated values from the regression model includes the support of the observed values.

2.3. Genetic Algorithm (GA)

GA is a stochastic approach based on the principle of "survival of the fittest" and "natural selection". GA belongs to the evolutionary algorithm family [17] applied to solve the optimization problems using the techniques based on natural evolution. In a complicated multi-dimensional search space, this algorithm is well-suited for finding the global optimal solution [18, 19]. The general GA procedure can be summarized as follows:

Step 1. The initial population is randomly generated and codified by chromosomes. They are represented as a vector of real numbers, of which, every entry is one of the unknown parameters of the problem.

Step 2. Each individual is evaluated in the population using a defined fitness function.

Step 3. In each iteration, each chromosome undergoes off-spring, cross-over, and mutation to produce a new population.

2.4. Adaptive Neuro-Fuzzy Inference System (ANFIS)

ANFIS is a famous hybrid technique that combines the adaptive learning capability of ANN along with the intuitive fuzzy logic of human reasoning. Thus the advantages of a fuzzy system can be combined with a learning algorithm [20, 21]. It is one of the most popular fuzzy neural systems based on the concepts of fuzzy if-then rules [22].

In order to present the ANFIS model architecture, we consider four fuzzy if-then rules with two input variables and one output variable.

Rule 1: If x_1 is A_1 and x_2 is A_3, then $f_1 = p_0^1 + p_1^1 x_1 + p_2^1 x_2$

Rule 2: If x_1 is A_1 and x_2 is A_4 then $f_2 = p_0^2 + p_1^2 x_1 + p_2^2 x_2$

Rule 3: If x_1 is A_2 and x_2 is A_3, then $f_3 = p_0^3 + p_1^3 x_1 + p_2^3 x_2$

Rule 4: If x_1 is A_2 and x_2 is A_4, then $f_4 = p_0^4 + p_1^4 x_1 + p_2^4 x_2$

where, x_1, x_2, and x_1, x_2 and $y \in R$ are the input and output variables, respectively, A_k's are the fuzzy sets, and f_k represents the system output due to rule $R_k\,(k = 1,2,3,4)$. In what follows, the five layers of the system that have 2D inputs and one output are explained. In the first layer, all the nodes are adaptive nodes that generate membership grades of the inputs. The node functions are given by:

$$o_{1,k} = \mu_{A,k}(x) \exp\left[-\left(\frac{x_{j,i} - \tau_k}{2\sigma_k} \right)^2 \right], \quad k = 1,2,3,4, i = 1,2, j = 1,..,n \tag{6}$$

$o_{l,k}$ is the output of the k^{th} node of layer l. In this work, the Gaussian membership function is considered. The τ_k and σ_k parameters represent the center and the spread, respectively.

In the second layer, the nodes are also fixed. In this layer, the outputs can be calculated as follows:

$$o_{2k} = w_{jk} = \mu_{Ak}(x_{j1}).\mu_{Ak} x_{j2}), \quad k = 1,2,3,4, j = 1,...,n.$$

In the third layer, the nodes are fixed nodes. The outputs of this layer can be calculated as:

$$o_{3k} = \bar{w}_{jk} = \frac{w_{jk}}{\sum_{k=1}^{4} w_{jk'}} \mu_{Ak}(x_{j1}).\mu_{Ak} x_{j2}), \quad k = 1,2,3,4 \quad j = 1,2,...,n \tag{7}$$

This is called the normalized firing strength.

In the fourth layer, the node is an adaptive one. The association node function in this layer is a linear function and the outputs can be represented as follows:

$$o_{4k} = \bar{w}_{jk} f_k = \bar{w}_{jk} p_i^k, \quad k = 1,2,3,4 \quad i = 1,2. \tag{10}$$

In this work, p_i^k is assumed to be a symmetric triangular fuzzy number.

In the fifth layer, the single node carries out the sum of the inputs of all the layers. The overall output of the structure can be expressed as:

$$o_{5k} = \sum_{k=1}^{4} \bar{w}_{jk} f_k \tag{11}$$

3. Methodology of proposed method

In (11), assume that $x_j = 1, x_{j1}, x_{j2}, ..., x_{jp}$ is a p-dimensional input vector of the independent variables at the j^{th} observation. Also $p = (p_0, p_1, ..., p_p)$ a vector of unknown fuzzy parameters, and $Y_i = (\alpha_{y,i}, \beta_{y,i})$ and $Y_j = (\alpha_{yj}, \beta_{yj})$ are the j^{th} observed value and the estimated value of the dependent variables for $j = 1, 2, ..., n$ where n is the number of data points, α_{yj} is the center value, β_{yj} is the spread value of Y_j, α_{yj} is the center value, and β_{yj} is the spread value of Y_j.

$p_i, i = 0, ..., p$ can be denoted in the vector form as $p_i = \{b, \alpha\}$ where $b = (b_0^k, b_1^k, ..., b_p^k)$ and $\alpha = (\alpha_0^k, \alpha_1^k, ..., \alpha_p^k)$ $k = 1, \ldots, m$, where b_i^k is the center value and α_i^k is the spread value of p_i, $i = 0, \ldots, p$ Thus from the above definitions, using the fuzzy arithmetic and substituting p_i^k into Eq. (11), it can be expressed as:

$$Y_j = \sum_{k=1}^{m}\sum_{i=0}^{p} b_i^k \bar{w}_{jk} x_{ji} + \sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_{jk} x_{ji} \tag{12}$$

where w_k is known.

3.1 Premise and consequence parameters

In the proposed methods, the linear programming method (forward pass) is used to optimize the consequent parameters. Once the optimal consequent parameters are found, the gradient descent (backward pass) is used to optimize the premise parameters. For more details, see [20, 22]. In the proposed methods, the premise parameters are obtained by the ANFIS method for only the first part of Eq. (12). In what follows, the parameters obtained are optimized by GA. Then the consequence parameters are obtained by solving the following LP model:

$$\min \sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_k x_{ji}$$

so that the following constraints must be established:

$$\begin{aligned}&\sum_{k=1}^{m}\sum_{i=0}^{p} b_i^k \bar{w}_k x_{ji} - (1-h)\sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_k x_{ji}\\ &\le a_{y_j} - (1-h)\beta_{y_j}\end{aligned} \tag{13}$$

$$\begin{aligned}&\sum_{k=1}^{m}\sum_{i=0}^{p} b_i^k \bar{w}_k x_{ji} + (1-h)\sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_k x_{ji}\\ &\ge a_{y_j} + (1-h)\beta_{y_j}\end{aligned} \tag{14}$$

and

$$\sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_k \ge 0, \quad i = 0, \ldots, p,$$
$$k = 1, \ldots, m, \quad j = 1, \ldots, n$$

3.2. Proposed methods for performance evaluation

In order to evaluate the performance of the various methods, an error rate can be calculated as follows [5]:

$$\begin{aligned}ERROR &= \frac{1}{n}\sum_{j=1}^{n}\left(Y_j - \hat{Y}_j\right) = \\ &\frac{1}{n}\sum_{j=1}^{n}\left(3\left(a_{y_j} - \sum_{k=1}^{m}\sum_{i=0}^{p} b_i^k \bar{w}_{jk} x_{ji}\right)^2 + 2\left(\beta_{y_j} - \sum_{k=1}^{m}\sum_{i=0}^{p} \alpha_i^k \bar{w}_{jk} x_{ji}\right)^2 \right)\end{aligned} \tag{15}$$

Equation (15) is used as a quantity to measure bias between $x_j s(j = 1, 2, ..n)$ where $l_j, a_j, r_j, \hat{l}_j, a_j, r_j$ are the lower, center, and upper of the observed fuzzy outputs and lower, center, and upper of the estimated the observed values, $Y_j = (l_j, a_j, r_j)$, and the predicted values, $Y_j = (\hat{l}_j, a_j, r_j)$, for all fuzzy outputs.

3.3. Learning algorithm for forecasting model

In order to forecast the model parameters, the steps taken can be summarized as follow:

Step 1: Divide all data into the two subsets of train and test data.

Step 2: Input value of α.

Step 3: Obtain the premise parameters by the ANFIS method.

Step 4: Put the premise and consequent parameters obtained in a matrix $p = xp_0$, where $x \in \left[10^{-7}, 10^{7}\right]$

Step 5: Optimize the premise parameters obtained by GA.

Step 6: Identify the consequent parameters by Eqs. (12), (13), and (14).

Step 7: Terminate the training of network when ERROR in Eq. (14) is smaller than a pre-defined small number or reaches a pre-defined epoch

number; otherwise, go to step 4 and update the premise parameters.
Step 8: Evaluate the chosen net using the test data.
In this work, we used the MATLAB software tool for codding.

4. Numerical examples

In order to demonstrate the applicability of the proposed algorithm, consider the following simulation and practical examples, and compare the results obtained from the different methods.

Example 1: Simulation example

Consider the following functions:

$$f(x_1,x_2)=24.23r^2(0.75-r^2)+5,$$
$$r^2=(x_1/10-0.5)^2+(x_2/10-0.5)^2.$$

Suppose that the domain of $X=(x_1,x_2)$ is $D=[0,10]^2$. A set of data is generated in the following manner.
The independent variables x_1 and x_2 are the crisp inputs. They are randomly taken from 0 to 10. Let the output $Y_j=(a_j,\beta_j)$ ($j=1,2,\dots,n$) be a symmetric fuzzy number; it is generated as follows:

$$\begin{cases} a_j=f(x_{j1},x_{j2}), \\ \beta_j=(1/4)f(x_j)+rand[0,1], \end{cases} \quad j=1,\dots,30,$$

where rand $[a,b]$ denotes a random number between a and b for each j. We apply the different methods for fitting the regression model and use the error value ERROR numerically to evaluate the performance of the different methods. In what follows, the parameters obtained for using the FWLP method and the proposed method are, respectively, displayed in tables 1 and 2. Also the regression model obtained in the LP method is as follows:

$$\hat{Y}_j=(\hat{a}_j,\hat{\beta}_j)=(0.0744,2.7036)+$$
$$(1.4498,0.2699)X_{j1}+(0.4007,0.1173)X_{j2}$$

In tables 3 and 4, the results obtained for the different methods are summarized. It can be observed that the error value of the proposed method is lower than the error values of the other ones.

Table 1. Premise and consequence parameters obtained using FWLP method.

Variable	k	(τ_k,σ_k)	(b_0^k,α_0^k)	(b_1^k,α_1^k)	(b_2^k,α_2^k)
x_1	1	(4.5057,1.5576)	(5.3794,0.7065)	(1.6089,0.4154)	(-0.2687,0.2711)
	2	(8.3810,2.3388)	(-1.5851,0.7099)	(1.8465, 0.5218)	(0.6591,0.0417)
x_2	3	(6.4446,2.8907)	(1.0780,0.5292)	(1.6831,0.4525)	(0.0664,0.0352)
	4	(3.6626,1.9964)	(-24.3688,0.8157)	(4.9530, 0.2096)	(-0.819, 0.2715)

Tables 2. Premise and consequence parameters obtained using FWGALP method.

Variable	k	(τ_k,σ_k)	(b_0^k,α_0^k)	(b_1^k,α_1^k)	(b_2^k,α_2^k)
x_1	1	(2.5012,0.5197)	(6.5359, 0.1967)	(-3.5306, 0.1750)	(2.2994, 0.6365)
	2	(0.0044,1.6937)	(-4.9901, 0.9352)	(3.9159, 0.2645)	(-0.0074, 0.0939)
x_2	3	(5.4523,0.9135)	(2.1379, 0.1523)	(1.3080, 0.5804)	(0.0867, 0.0869)
	4	(0.5245,9.9232)	(-0.7306, 1.6641)	(1.5325, 0.4262)	(0.4663, 0.0020)

Table 3. Predicted fuzzy outputs using the different methods for Example 1.

k	x_1	x_2	$Y_j=(a_j,\beta_j)$	$\hat{f}(x_j)=(\hat{a}_j,\hat{\beta}_j)$ LP	$\hat{f}(x_j)=(\hat{a}_j,\hat{\beta}_j)$ FWLP	$\hat{f}(x_j)=(\hat{a}_j,\hat{\beta}_j)$ FWGALP
1	6.6280	6.6090	(11.6800,3.6280)	(12.3319,5.2680)	(11.6917,4.2428)	(12.1123,4.5273)
2	9.2970	7.1810	(16.9620,4.8400)	(16.4306,6.055`5)	(16.9672,4.9535)	(16.5294,5.7326)
3	3.1780	7.1450	(7.7340,2.4680)	(7.5450,4.3997)	(8.0370,3.5867)	(7.9595,2.9660)
4	1.1940	6.2100	(3.9620,1.2360)	(4.2940,3.7545)	(4.5495,2.4290)	(4.2726,1.8809)
5	3.9220	8.9840	(9.7070,3.1400)	(9.3606,4.8162)	(9.0832,4.4044)	(9.6829,3.2162)
6	2.6390	5.7270	(7.2780,1.8480)	(6.1953,4.0878)	(6.7819,2.9067)	(7.5075,3.0955)
7	9.7360	8.8430	(17.8320,4.4820)	(17.7331,6.3690)	(17.9882,5.2545)	(18.3095,5.8321)
8	3.7110	3.8310	(6.9900,1.9900)	(6.9897,4.1547)	(7.2932,2.9339)	(7.4215,2.8538)
9	2.8300	3.6310	(5.8000,2.2370)	(5.6323,3.8935)	(5.8992,2.5099)	(5.7987,2.2953)
10	7.1880	1.7980	(11.4750,3.3530)	(11.2159,4.8547)	(11.4573,3.3884)	(11.1235,4.7309)
11	6.3490	2.3940	(9.5760,3.2770)	(10.2383,4.6982)	(9.5565,3.4622)	(10.1175,4.3734)
12	1.3000	4.8520	(4.1170,1.7250)	(3.9034,3.6237)	(4.0000,2.0636)	(4.0663,1.8703)
13	0.5160	5.9760	(3.2490,1.0740)	(3.2172,3.5440)	(3.1623,2.0436)	(3.0905,1.4378)
14	7.5790	3.5590	(12.3730,3.2320)	(12.4884,5.1669)	(12.1927,3.7952)	(12.5236,4.8967)
15	1.0710	7.4820	(4.4630,1.7000)	(4.6254,3.8705)	(4.6572,2.8348)	(4.2931,2.0707)
16	7.1060	2.0840	(11.2450,2.9840)	(11.2116,4.8661)	(11.1546,3.4413)	(11.1315,4.6963)
17	5.2480	6.6670	(10.1170,3.2140)	(10.3545,4.9023)	(10.4973,4.0892)	(10.1407,3.8687)
18	2.9670	8.8280	(6.7840,2.2730)	(7.9135,4.5402)	(7.6386,4.0518)	(6.9088,2.6502)
19	3.6090	8.8930	(8.9790,2.8670)	(8.8703,4.7211)	(8.6220,4.2868)	(9.0138,2.9753)
20	3.9190	8.9900	(9.3860,2.5520)	(9.3586,4.8161)	(9.0778,4.4053)	(9.6804,3.2140)
21	5.6960	9.6320	(12.0780,3.0950)	(12.1921,5.3711)	(11.6004,4.7585)	(12.4900,4.1110)
22	3.1610	7.1270	(8.0850,2.6180)	(7.5131,4.3930)	(8.0091,3.5741)	(7.9339,2.9685)
23	5.8020	0.2170	(8.9130,2.8190)	(8.5729,4.2951)	(8.8643,3.0698)	(8.2620,4.1371)
24	0.8070	9.8370	(4.8600,1.5760)	(5.1863,4.0755)	(3.9515,3.4784)	(5.0551,2.0281)
				Test data		
1	6.3490	2.6600	(10.1150,2.8770)	(10.3449,4.7294)	(9.5709,3.5088)	(10.2437,4.3725)
2	6.3490	2.3940	(9.5760,3.2770)	(10.2383,4.6982)	(9.5565,3.4621)	(10.1176,4.3736)
3	5.2480	6.6670	(10.1170,3.2140)	(10.3545,4.9023)	(10.4972,4.0890)	(10.1406,3.8686)
4	2.9670	8.8280	(6.7840,2.2730)	(7.9135,4.5402)	(7.6383,4.0517)	(6.9089,2.6504)
5	3.9190	8.9900	(9.3860,2.5520)	(9.3586,4.8161)	(9.0775,4.4051)	(9.6803,3.2140)
6	5.6960	9.6320	(12.0780,3.0950)	(12.1921,5.3711))	(11.6002,4.7583)	(12.4898,4.1110)

Tables 4. Error results obtained using the different methods for Example 1.

Different methods	Value ERROR of train	Value ERROR of test
LP	8.2812	8.8349
FWLP	2.6819	4.2682
FWGALP	1.9167	2.1179

Example 2: Prediction of diametral error in turning process

During turning the slender parts, the elastic deformation of the workpiece affects the dimensional accuracy of the workpiece [23-25]. A slender workpiece that clamped only in the chuck can be structurally regarded as a simple beam (Figure 1). As shown in this figure, the maximum deflection occurs at the free end of the workpiece, where the radial cutting force is acted. As a result of this deflection, the workpiece will have a non-uniform diameter, and the maximum deflection error occurs at the unsupported end of the workpiece (Figure 2).

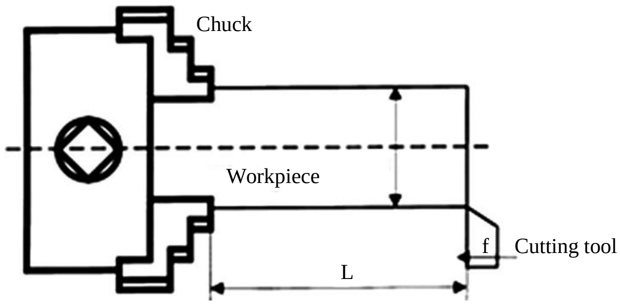

Figure 1. Workpiece clamped by a chuck in turning [26].

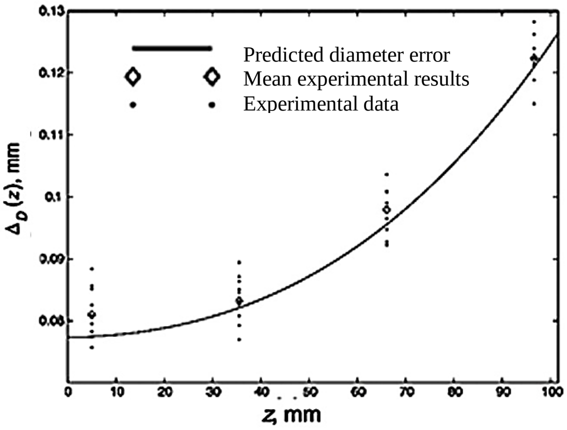

Figure 2. Diametral error of aworkpiece clamped by a chuck in turning [27].

Figure 3 illustrates the diameter measurement positions along the workpiece, conducted by Bendaros *et al*. [23] after the longitudinal turning of a cylindrical part. Four different measurement positions along the axis of the workpiece were chosen at distances of 20, 70, 100, and 150 mm from the free end of the workpiece. Due to the uncertainty and non-uniformity associated with the diameter at each measurement position, three different diameter measurements were taken. Note that in table 6, the different observed measurements at each position are indicated as the fuzzy outputs.

Table 5. Premise and consequence parameters obtained using FWLP method for Example 2.

k	(τ_k, σ_k)	(b_0^k, α_0^k)	(b_1^k, α_1^k)
1	(42.7171, 58.0730)	(51.1208, 0.0068)	(0.1689,0.0001)
2	(149.2036, 52.7325)	(-0.8580, 0.0060)	(0.3229, 0.0001)

Table 6. Premise and consequence parameters obtained using FWGALP method for Example 2.

k	(τ_k, σ_k)	(b_0^k, α_0^k)	(b_1^k, α_1^k)
1	(149.0171,12.2585)	(0.3449 ,0.0171)	(0.3445,0.0001)
2	(29.1182,54.1466)	(52.0146 ,0.0163)	(-0.0003,0.0002)

Table 7. Measured workpiece diameter and fuzzy outputs obtained using different methods for Example 2.

K	x_2	Diameter $Y_j = (a_j, \beta_j)$	$\hat{f}(x_j) = (\hat{a}_j, \hat{\beta}_j)$ FWLP method	$\hat{f}(x_j) = (\hat{a}_j, \hat{\beta}_j)$ FWGALP method
1	20	(52.0085,0.005)	(52.0087,0.0088)	(52.0082,0.0209)
2	70	(52.0050,0.004)	(52.0059,0.0136)	(51.9920,0.0324)
3	150	(52.0120,0.006)	(52.0141,0.0211)	(52.0120,0.0326)
1	100	(52.0120,0.001)	(49.2536,0.0164)	(51.9687,0.0393)

Table 8. Error results obtained using different methods for Example 2.

Different methods	Value ERROR of train	Value ERROR of test
FWLP	2.2844e-04	22.8271
FWGALP	0.0013	0.0086

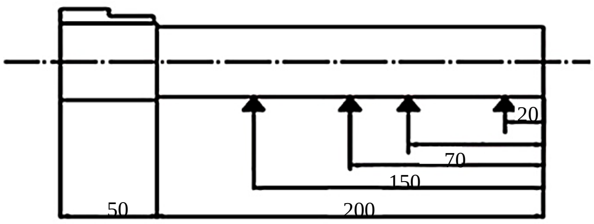

Figure 3. Diameter measurement positions after longitudinal turning of the workpiece [23].

In order to fit the regression model for diameter measurements (Table 6), we apply the different methods. For the performance evaluation of the different methods, we use the ERROR value. The premise and the consequence parameters obtained are shown in tables 5 and 6. In what follows, we calculate the output $\hat{Y} = (\hat{a}, \hat{\beta})$ for x = 100 in the test dataset. For the FWGALP method, $\hat{Y}$ can be calculated as follows:

At first, w_k is calculated for $k = 1,2$. Using the premise parameters in table 6, and Eqs. (6) and (9), and $x = 100$, w_k is equivalent to:

$$w_1 = \exp\left[(-1/2)\left(\frac{100 - 149.0171}{12.2585}\right)^2\right]$$

$$= 3.3732e - 04, w_2 = 0.4245, w_2 = 0.4245$$

and:

$$\sum_{k=1}^{2} w_{18,k} = 3.3732e-04+0.4245 = 0.4248$$

Therefore,

$$\bar{w}_1 = \frac{3.3732e-04}{0.4248} = 7.9399e-04$$

$$\bar{w}_2 = \frac{0.4245}{0.4248} = 0.9992,\ \bar{w}_2 = \frac{0.4245}{0.4248} = 0.9992$$

Finally, by substituting $\bar{w}_k$ and the obtained consequence parameters $(b_i^k, \alpha_i^k, k = 1,2, i = 0,1)$ (Table 6) in Eq. (12), $\hat{Y}$ is computed as follows:

$$\hat{a} = (7.9399e-04)(0.3449) +$$
$$(7.9399e-04)(0.3445)(100) +$$
$$(0.9992)(52.0146) + (0.9992)(-0.0003)(100)$$
$$= 51.8687$$

and:

$$\hat{\beta} = (7.9399e-04)(0.0171) +$$
$$(7.9399e-04)(0.0001)(100) +$$
$$(0.9992)(0.0163) + (0.9992)(0.0002)(100)$$
$$= 0.0393$$

Thereupon,

$$\hat{Y} = (\hat{a}, \hat{\beta}) = (51.8687, 0.0393)$$

Also in the FWLP method, $\hat{Y}$ is calculated as the FWGALP method.

The obtained results for the different methods are summarized in tables 7 and 8. Like the previous example, the proposed algorithm provides a better prediction than the FWLP method.

5. Conclusion

In this paper, we proposed an algorithm based on the neuro-fuzzy system, genetic algorithm, and linear programming (LP) in order to predict the fuzzy regression model. Also we used the simulation and case study examples to illustrate the applicability of the proposed algorithm in the case of crisp inputs and fuzzy output. We compared the results obtained for the different forecasting techniques. Using these results, a guideline could be proposed for selecting the appropriate regression method for predictive purposes. The main findings of this paper can be summarized as follow:

(1) Using the tables and the results obtained, we can see that the proposed method is stable. Based on the examples, the proposed method decreases the error values to a minimum level and is more accurate than the linear programming (LP) and fuzzy weights with linear programming (FWLP) methods.

(2) In the proposed method, the constrains used assure that the support of the estimated values from the regression model includes the support of the observed values in the h-level $(0 < \mathrm{h} \leq 1)$. To sum up, in LP, the width of the estimated value depends on the number of observations. As the number of observations increases, the width of the estimated value decreases.

(3) The proposed method is not more complicated than the FWLP method in computations but is more accurate.

(4) The results obtained from the simulated example and the case study in the field of turning process show that the presented method is especially useful for practical problems, which involve some degree of uncertainty, inhomogeneity, randomness, and imprecision in the output data.

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نشریه هوش مصنوعی و داده کاوی

الگوریتمی ترکیبی جهت برآورد پارامتر فازی با کاربرد در اندازه‌گیری‌های غیرقطعی

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چکیده:

در این مقاله روشی جدید برای پیش‌بینی مدل رگرسیون در محیط غیر قطعی ارائه شده است. در مسائل مهندسی کاربردی، به منظور ایجاد مدل رگرسیونی یا شبکه عصبی مصنوعی، مجموعه‌ای از میانگین‌های مقادیر مشاهده‌شده تکراری به عنوان متغیر ورودی جهت ایجاد مدل پیش بینی کننده، به مدل معرفی می شوند. در نتیجه، پا سخ برآورد شده نیز مجموعه‌ای از میانگین مقادیر خروجی خواهد بود که در آن تغییرات حول میانگین مـشخص نیست. به منظور بدست آوردن برآوردهای دقیق و غیر منحرف، پیش بینی‌ها باید از نظر میانگین و پراکندگی داده‌ها کاملاً مشخص باشد. به منظور حل این مـساله، رو شی مبتنی بر سیـستم ا ستنتاج فازی ارائه شده ا ست. داده‌های ورودی فازی و داده‌های خروجی فازی ذوزنقه‌ای متقارن در نظر گرفته و الگوریتم پیشنهادی جهت برازش مدل رگسیون فازی استفاده شد. علاوه بر این به منظور ارزیابی عملکرد روش پیشنهادی در مواجهه با مسائل کاربردی که در آنها متغیرهای خروجی دارای طبیعت غیر قطعی و غیردقیق هستند یک مثال شبیه‌سازی و یک مثال کاربردی در زمینه فرآیند ماشینکاری مورد ا ستفاده قرار گرفت. در پایان، عملکرد روش پیـشنهادی با سایر روش‌های موجود مقایـسه شد. بر ا ساس مثال‌های به‌کار رفته، روش پیـشنهادی جهت پیش‌بینی تصدیق شد. نتایج بدست آمده نشان داد روش پیشنهادی نسبت به روش برنامه ریزی خطی و روش برنامه ریزی خطی با وزن‌های فازی (FWLP) دقت بیشتری دارد.

کلمات کلیدی: رگرسیون فازی، برنامه‌ریزی خطی، فرآیند ماشین‌کاری، سیستم استنتاج عصبی فازی تطبیق، الگوریتم ژنتیک.

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